IEEE Ultrasonic symposium 2002

Short Course 6: Flow Measurements

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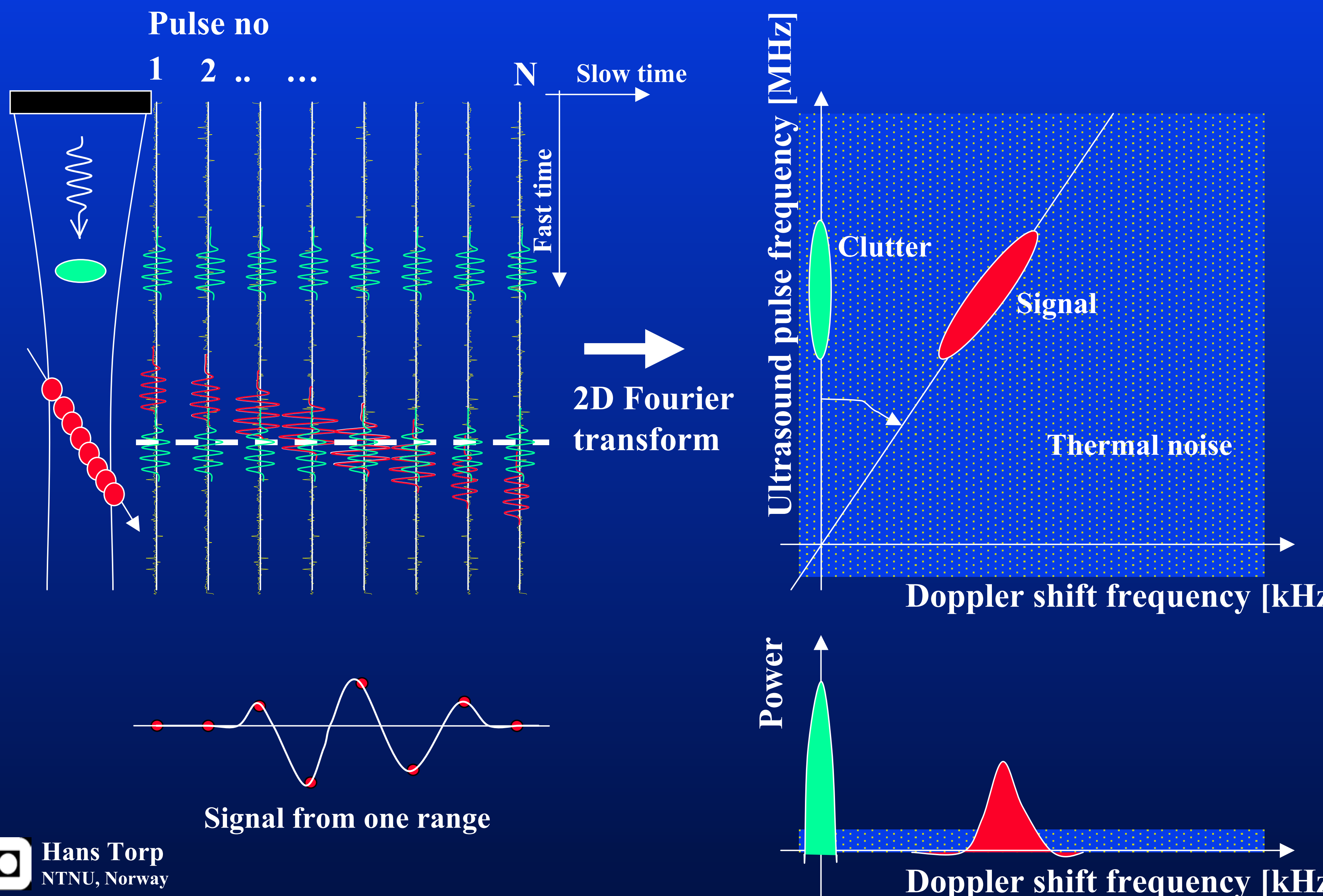
Internet-site for short course:
http://www.ifbt.ntnu.no/~hanst/flowmeas02/index.html

Lecture 4: Advanced methods

Signal from moving scatterer



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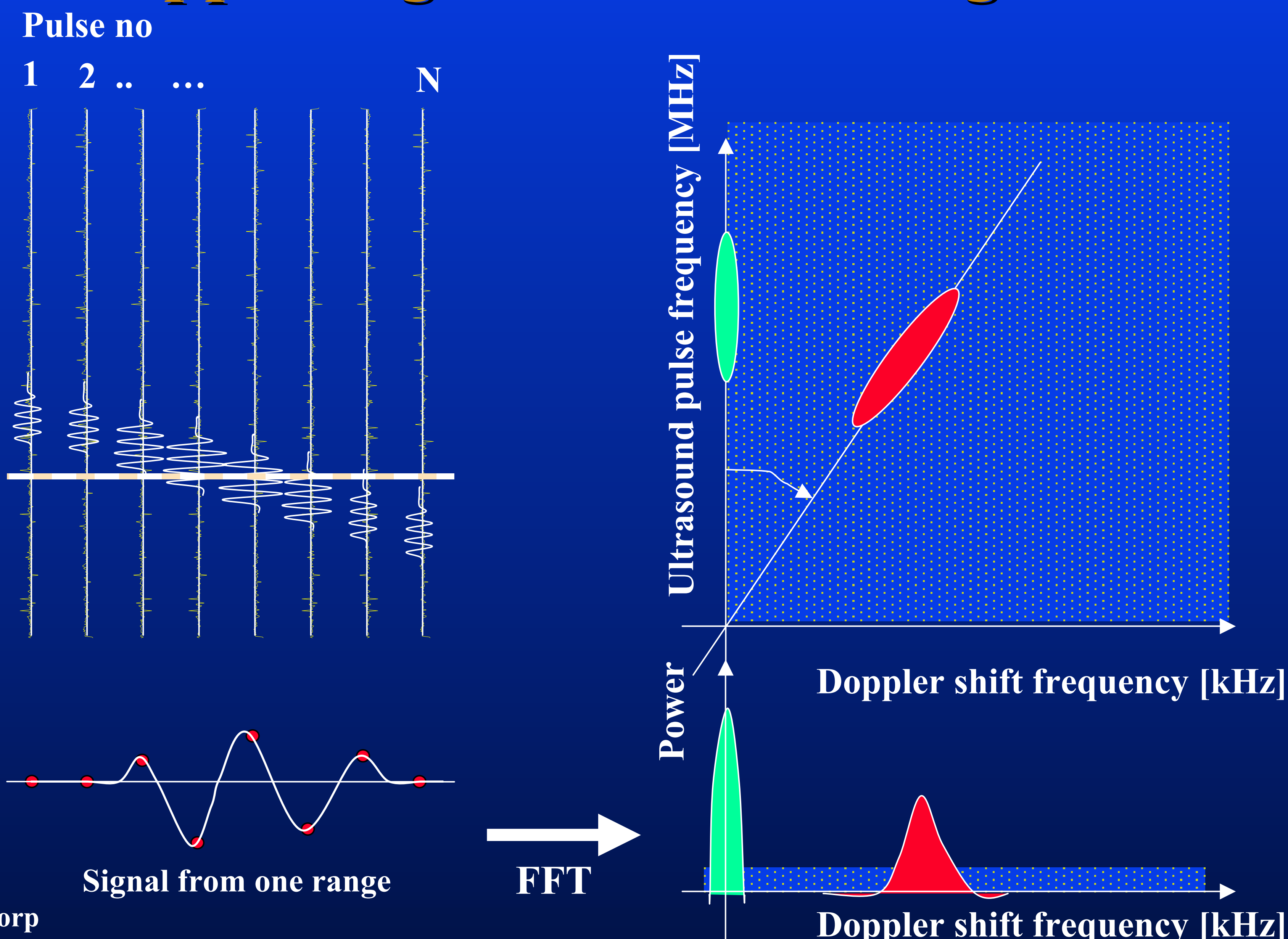
Go to lecture 1!

Doppler signal from one range



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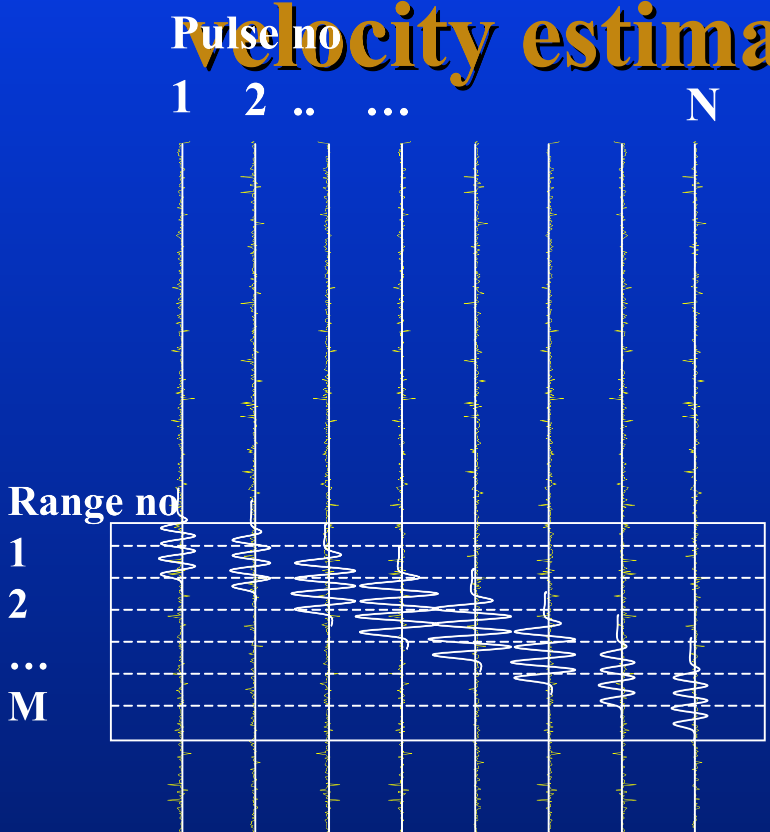
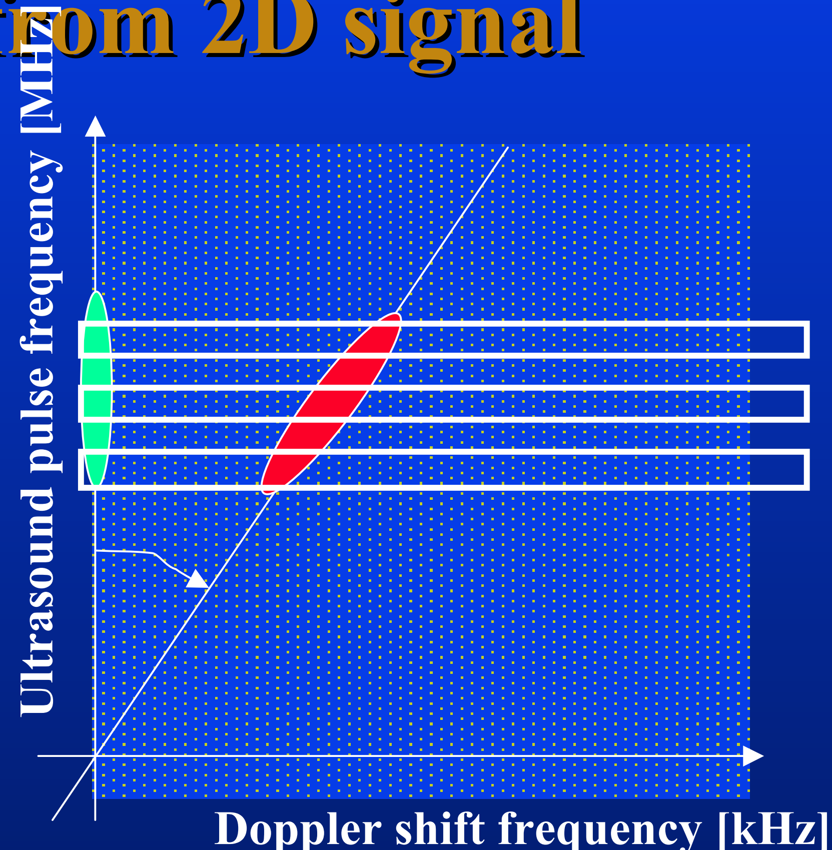
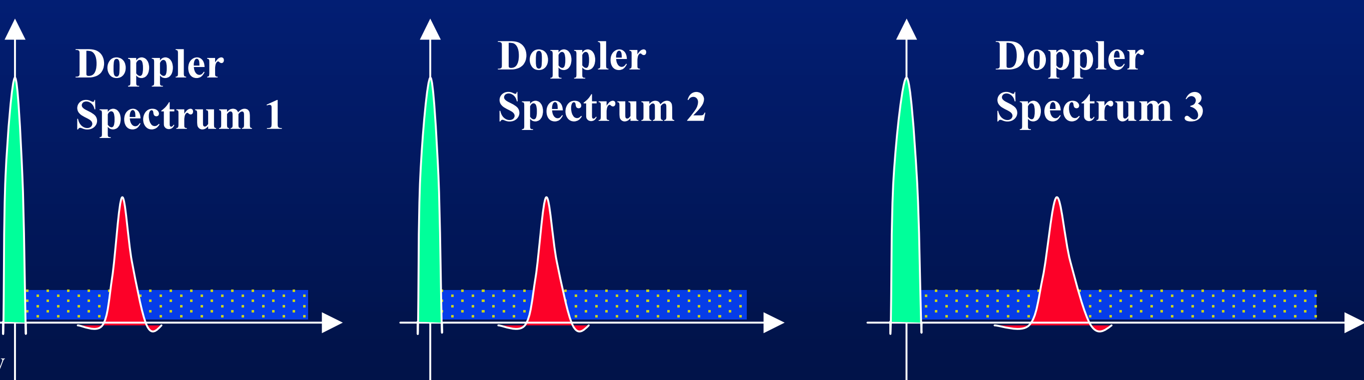
Blood detection and velocity estimation from 2D signal







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Blood detection and velocity estimation from 2D signal

- Increased number of range samples M give better performance but lower spatial resolution
- Best spatial resolution with M=1
- In this work optimum estimators for the case M=1 is treated
- Extension to the case M > 1 is straight forward

1D Signal Model

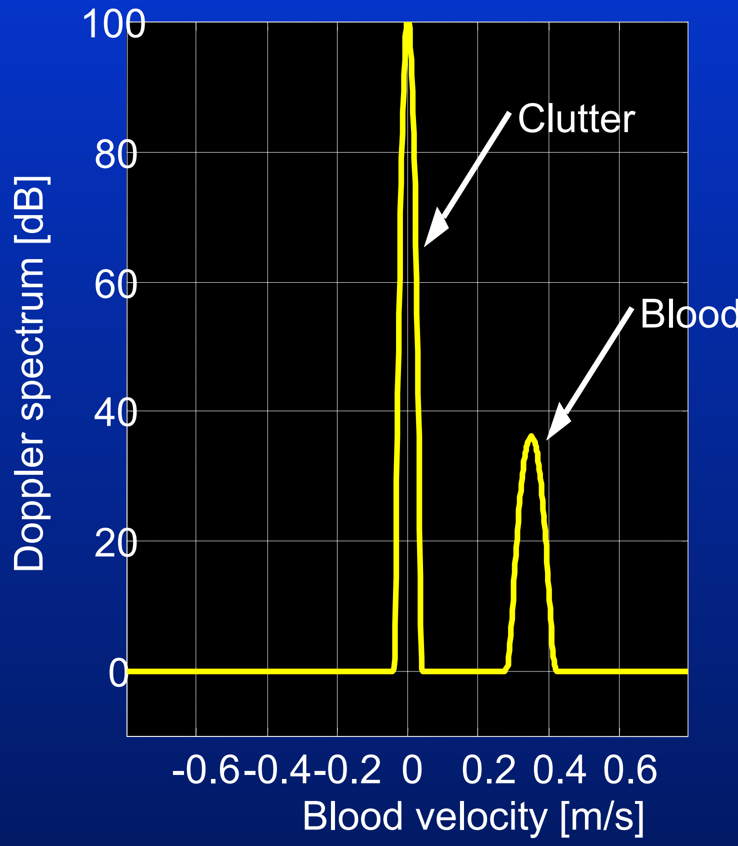
- Signal vector for each sample volume:

$$\mathbf{x} = [x(1),\ldots,x(\mathrm{N})]^{\mathrm{T}}$$

- Zero mean complex Gaussian process
- Three independent signal components:

Signal = Clutter + White noise + Blood

$$\mathbf{x} = \mathbf{c} + \mathbf{n} + \mathbf{b}$$



- Signal correlation function:

$$R_x(m) = \left\langle x(k)^* x(k+m) \right\rangle = R_c(m) + \delta(m) + R_b(m)$$

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Probability density function for the complex signal vector x

$$p(x|v) = \frac{1}{\pi^N |C(v)|} e^{-x'C^{-1}(v)x}$$

$$C(v) = \left\{R_x(k-n)\right\}_{k,n}$$

$$C(v) = C_c + I + C_b(v)$$

v = radial velocity component of blood flow

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Log likelihood function and Cramer - Rao lower bound

$$l(v|x) \equiv \log p(x|v) = -x^T C^{-1}(v) x - |C(v)|$$

$$\mathrm{var}_{\min} = \left\langle \frac{\partial^2}{\partial^2 v} l(v|x) \right\rangle^{-1}$$

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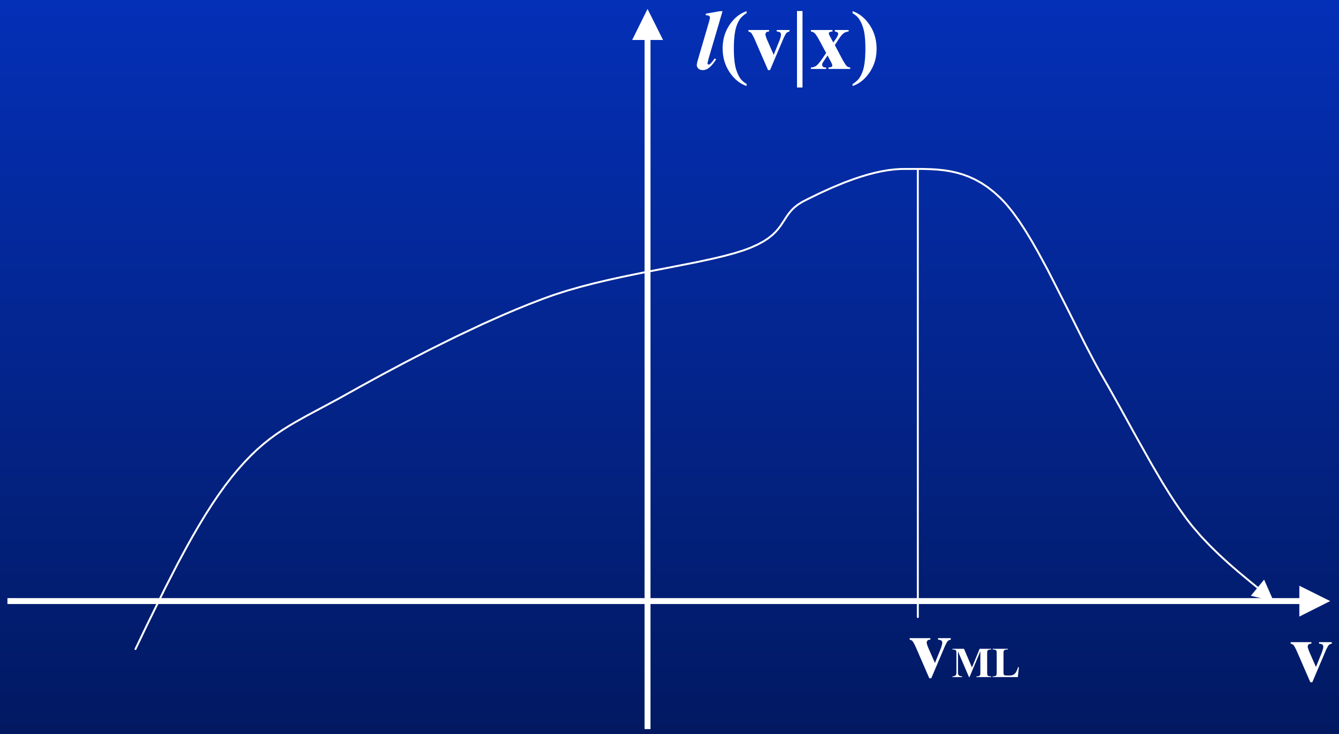
Maximum Likelihood estimator



Cramer - Rao lower bound Approximation

$$\left\langle \frac{\partial^2}{\partial^2 v} l(v|x) \right\rangle = -\left\langle x^T C^{-1}(v)''(v)x \right\rangle - \left|C(v)\right|''$$

$$\left|C(v)\right|'' \approx 0 \; ; \; R_b(m) = \left|R_b(m)\right| e^{i2mkTv}$$

$$\frac{\partial}{\partial v}\left|R_b(m)\right| \approx 0$$

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Cramer - Rao lower bound Approximation

$$\left\langle \frac{\partial^2}{\partial^2 v} l(v|x) \right\rangle \approx -\sum_{m,n} c(m,n)\, b(m,n)$$

$$C = \{c(m,n)\}_{m,n} = C_C + I + C_b$$

$$B = \{b(m,n)\}_{m,n} = C^{-1}\left(2C_b^{'}C^{-1}C_b^{'} - C_b^{''}\right)C^{-1}$$

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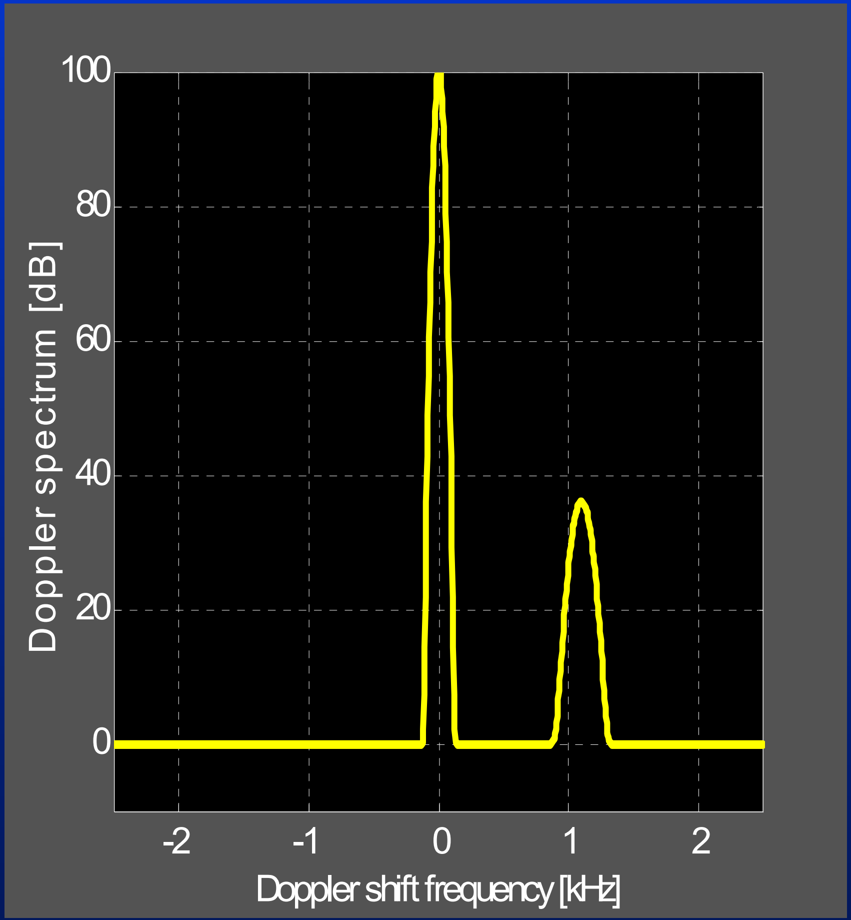
Signal simulation model



Frequency	2.5 MHz
Beam width	3 mm
Pulse length	2 mm
PRF	5 kHz
packet size	10 samples
Signal level	20 dB
Clutter level	80 dB
Blood velocity	0.2 -0.8 m/s
Angle blood flow	20 deg.

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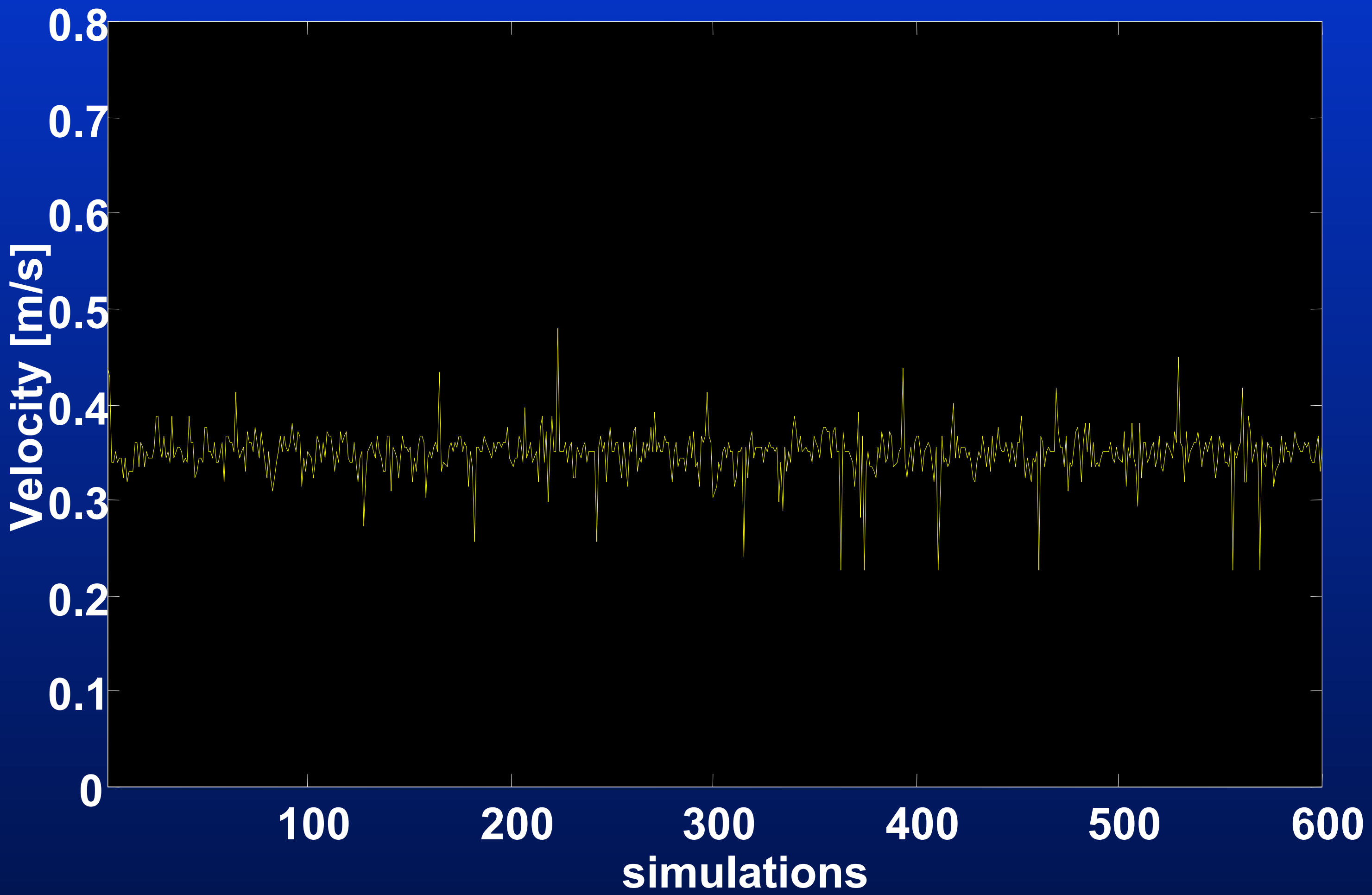
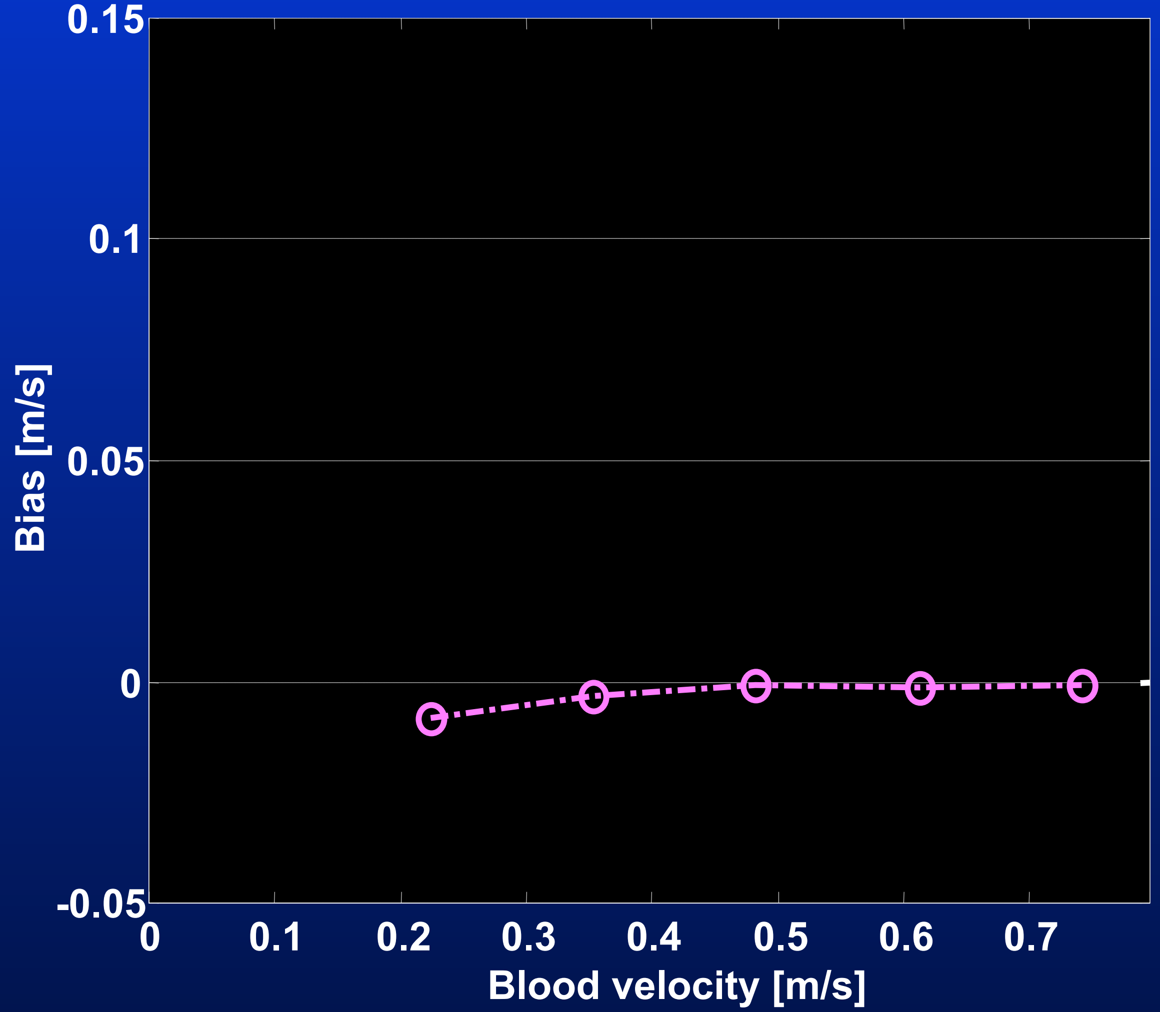
Numerical simulations maximum likelihood estimator



Maximum likelihood estimator bias



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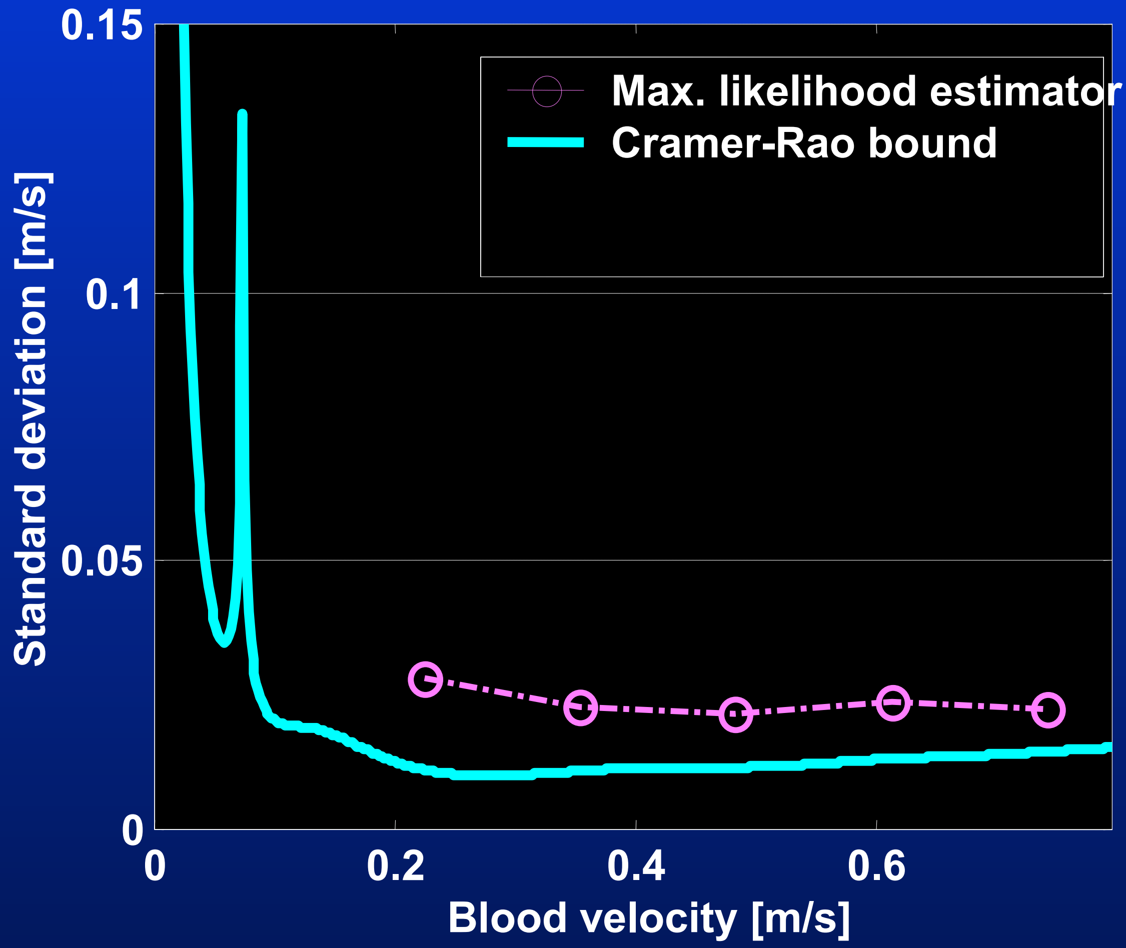
Maximum likelihood estimator properties



ML estimator is unbiased, but not efficient (minimum variance)

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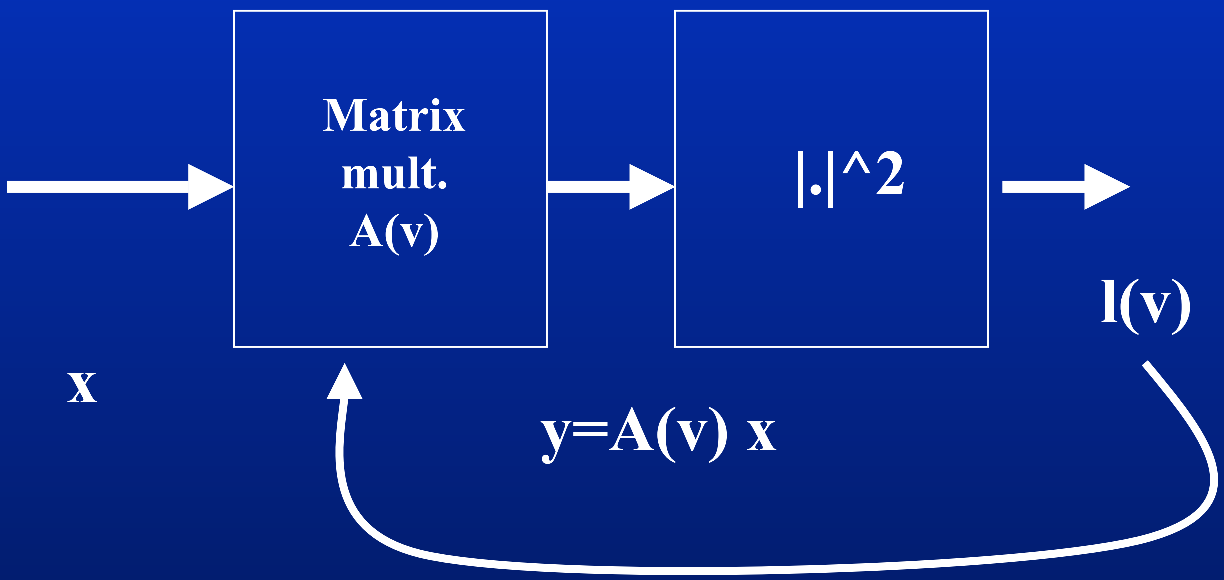
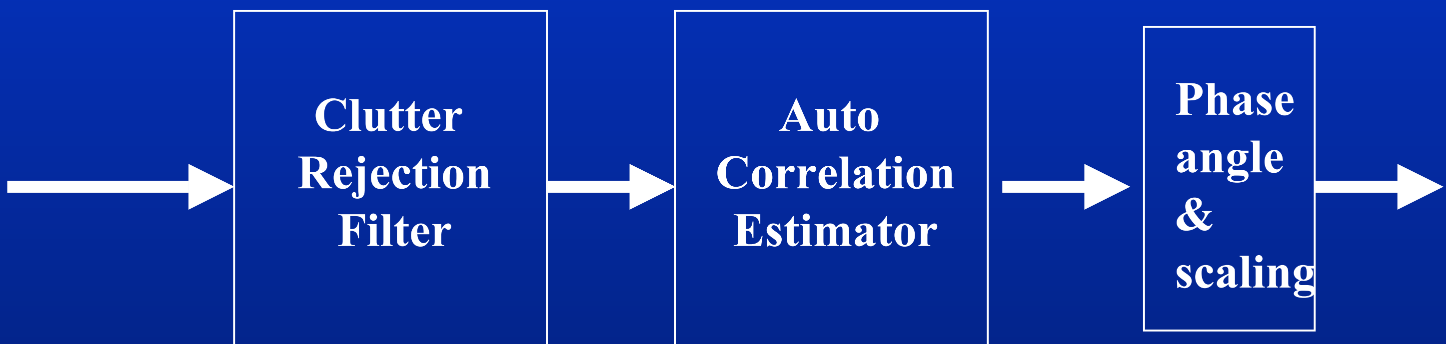
Max. likelihood method for blood velocity estimation



Autocorrelation method for blood velocity estimation



x **y=A x** $\hat{R}(1)$ $\hat{v} \sim angle(\hat{R}(1))$

$$= \sum_n y(n)^* y(n+1)$$

- **Clutter Rejection filter formulated as a matrix multiplication**
- **Includes FIR filter, initialized IIR filter, and regression filter**

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FIR filter

$$A = \begin{bmatrix} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & 0 & . & . & 0 \\ 0 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & & & . \\ . & & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & & . \\ . & & & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & 0 \\ 0 & . & . & 0 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \end{bmatrix}$$

FIR filter order M=5
Packet size N=10
Output samples: N-M= 5

Increasing filter order

+ Improved clutter rejection

- Increased estimator variance

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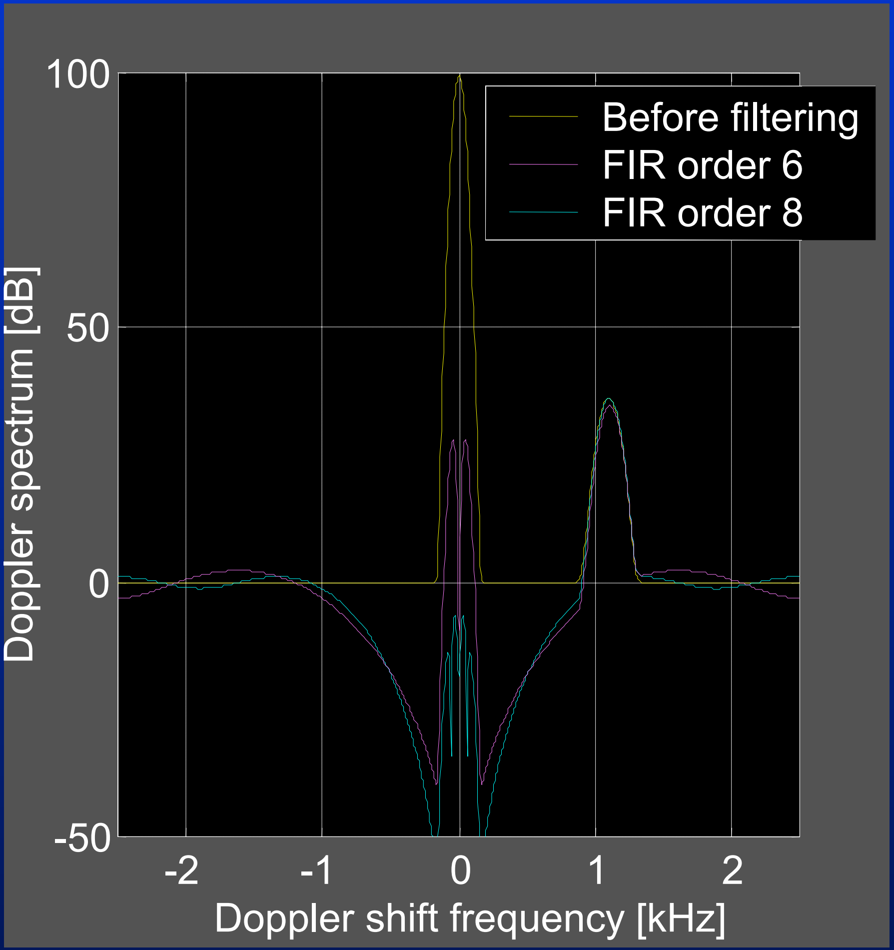
Clutter suppression by high pass filtering



Packet size N=10

Order M=6: 4 samples left after initialization

Order M=8: 2 samples left after initialization

FIR filter

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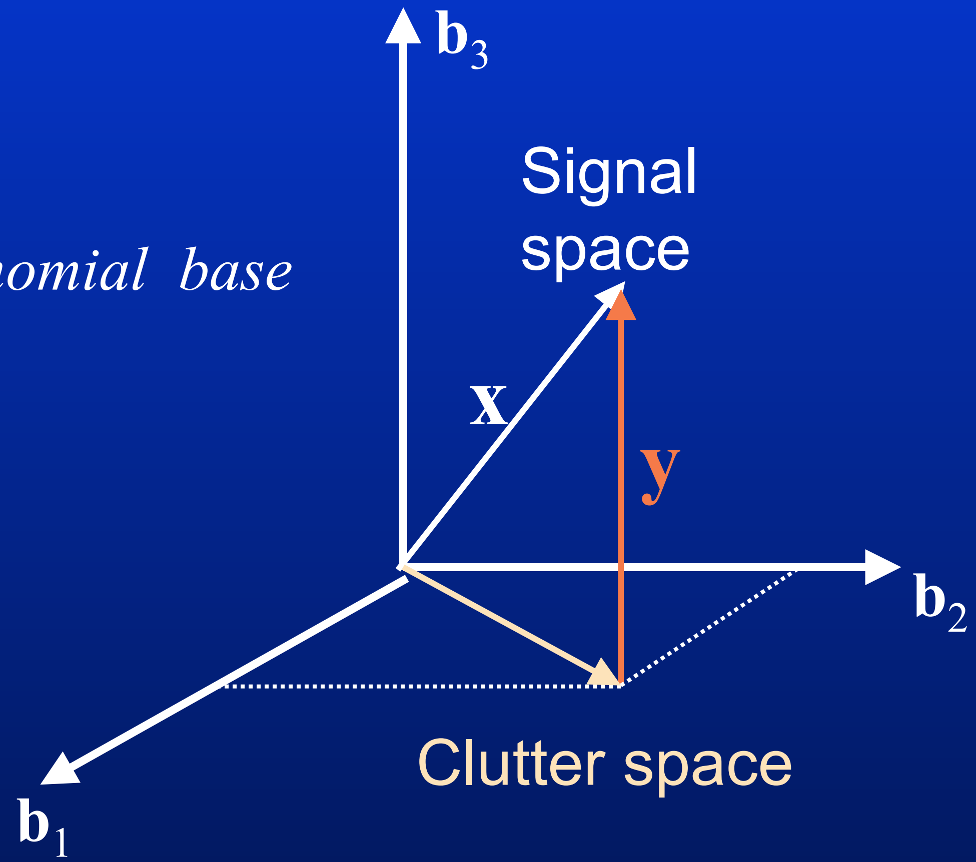
Polynomial regression filter

$$A = I_N - \sum_{k=0}^{P} b_k \, b_k^{\,T}$$

$$b_0, b_1, .., b_{N-1} \quad Legendre \ polynomial \ base$$

- **Subtraction of the signal component contained in a *P*-dimensional clutter space:**



Increasing polynomial order → **Increasing cut off frequency**

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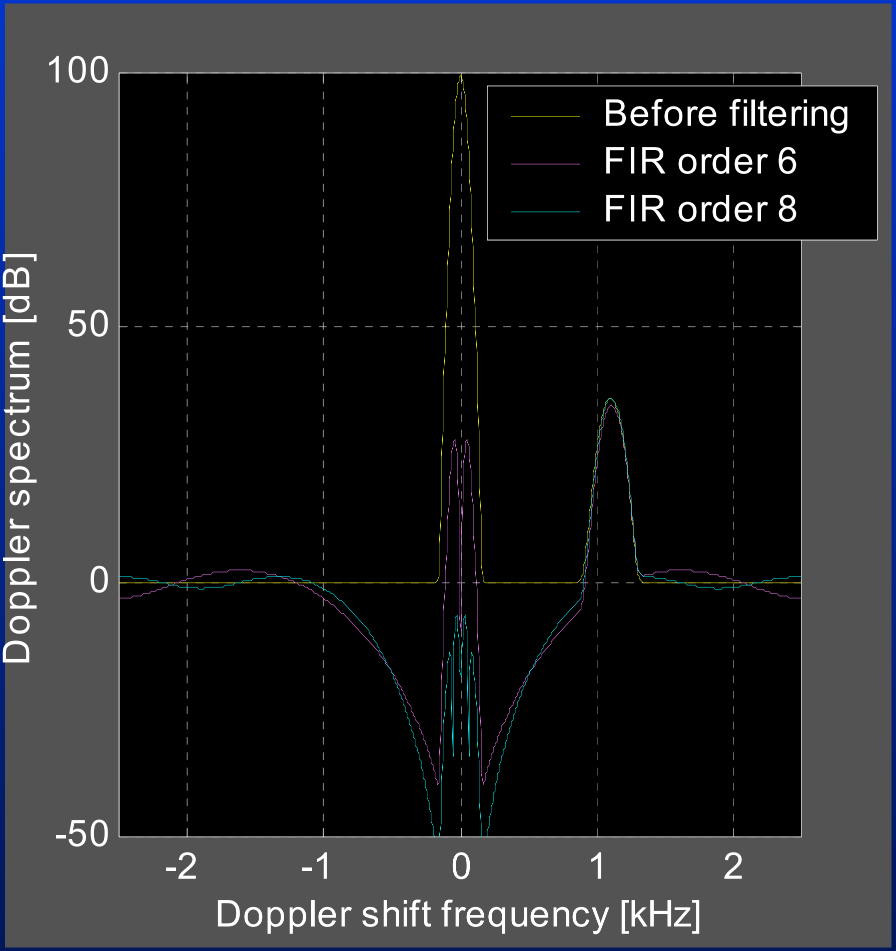
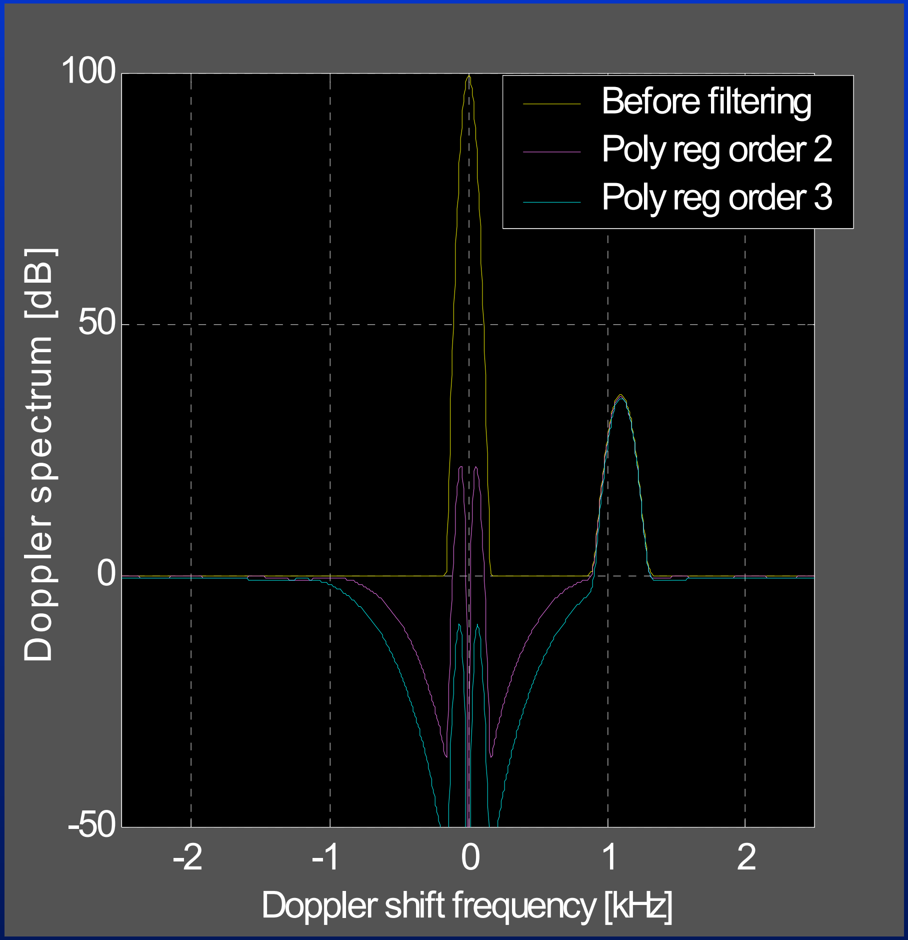
Clutter suppression by high pass filtering





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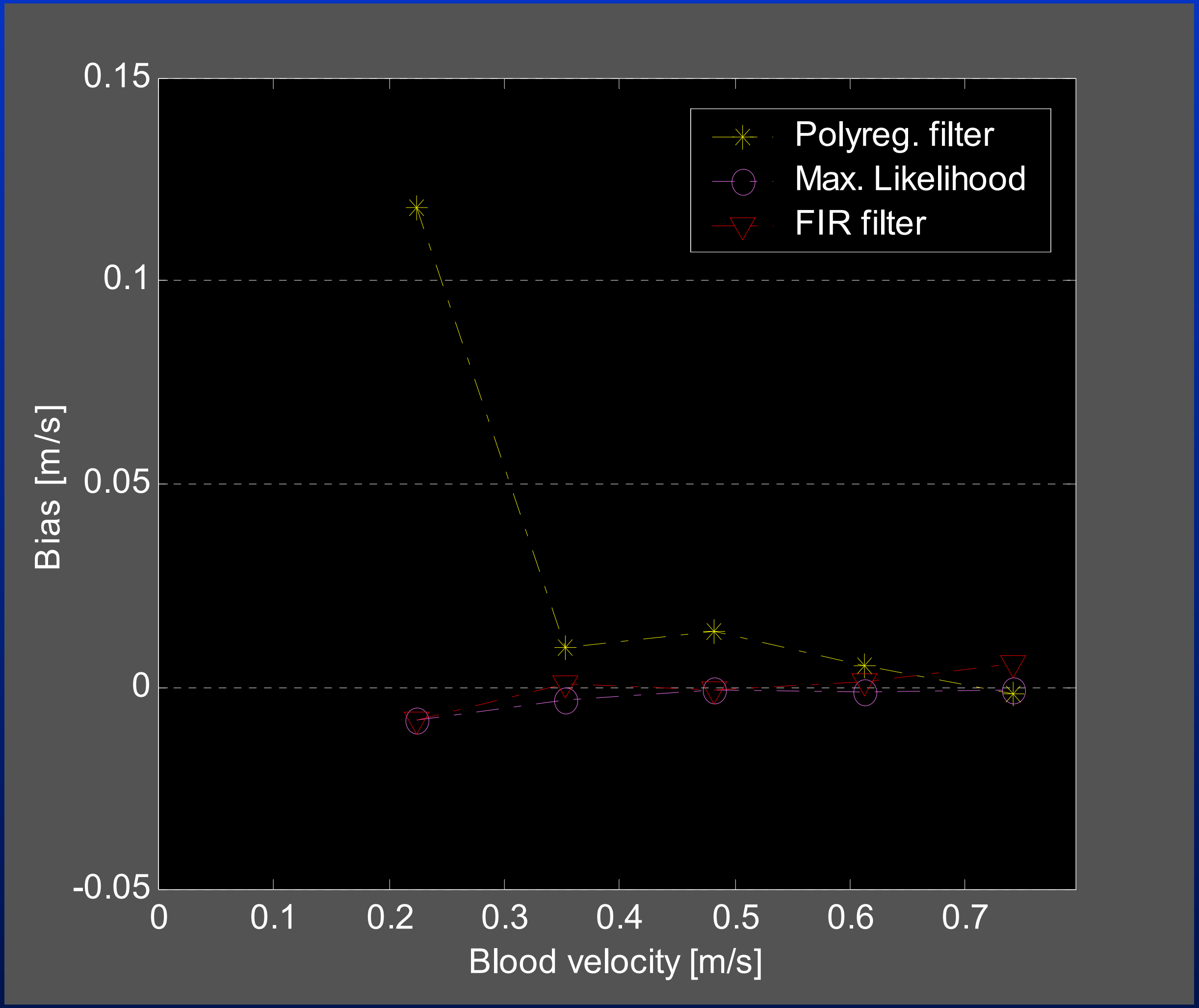
FIR filter

Polynom regression filter

Blood velocity estimator bias



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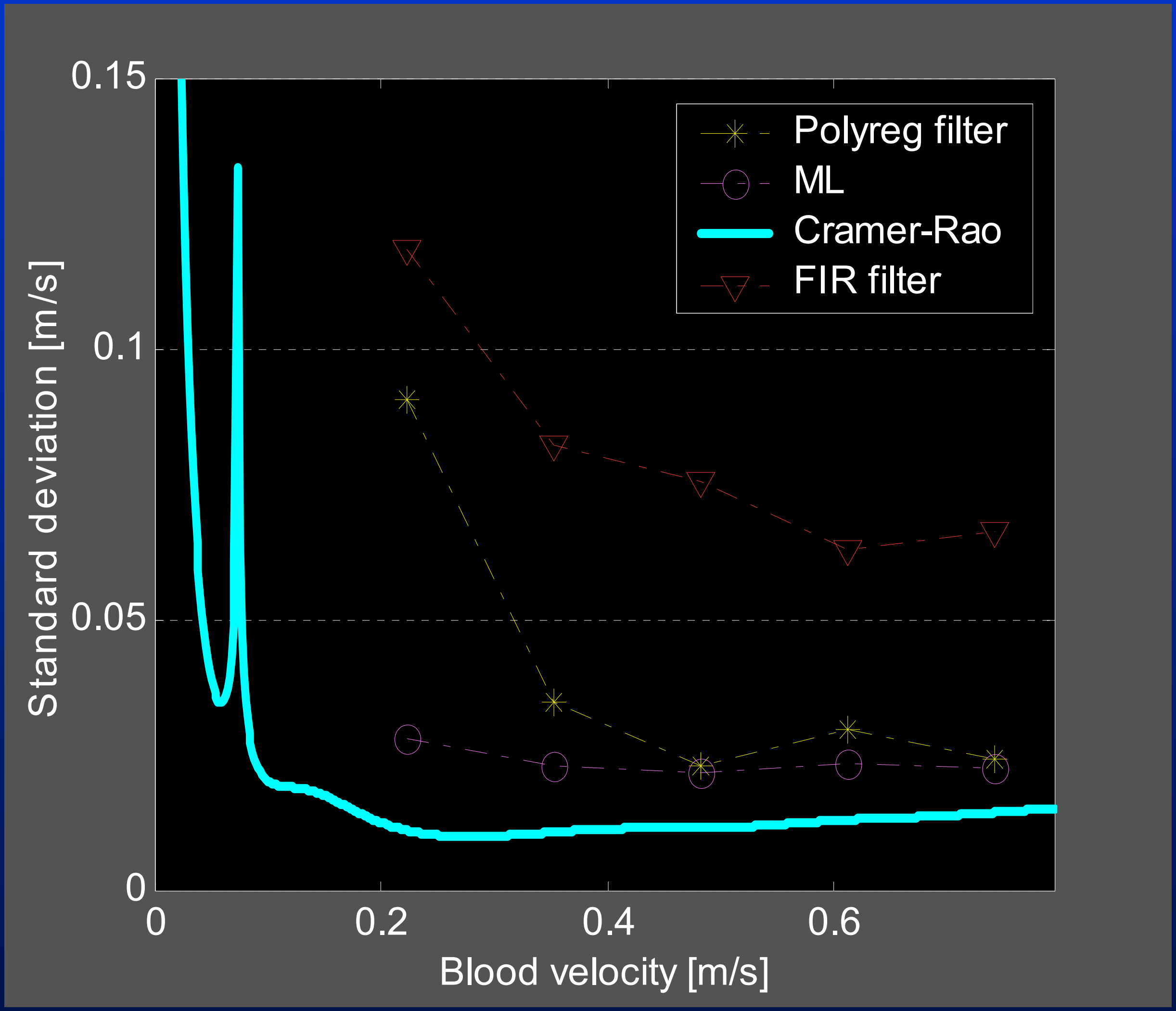
Blood velocity estimator variance



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Conclusion Optimum velocity estimation

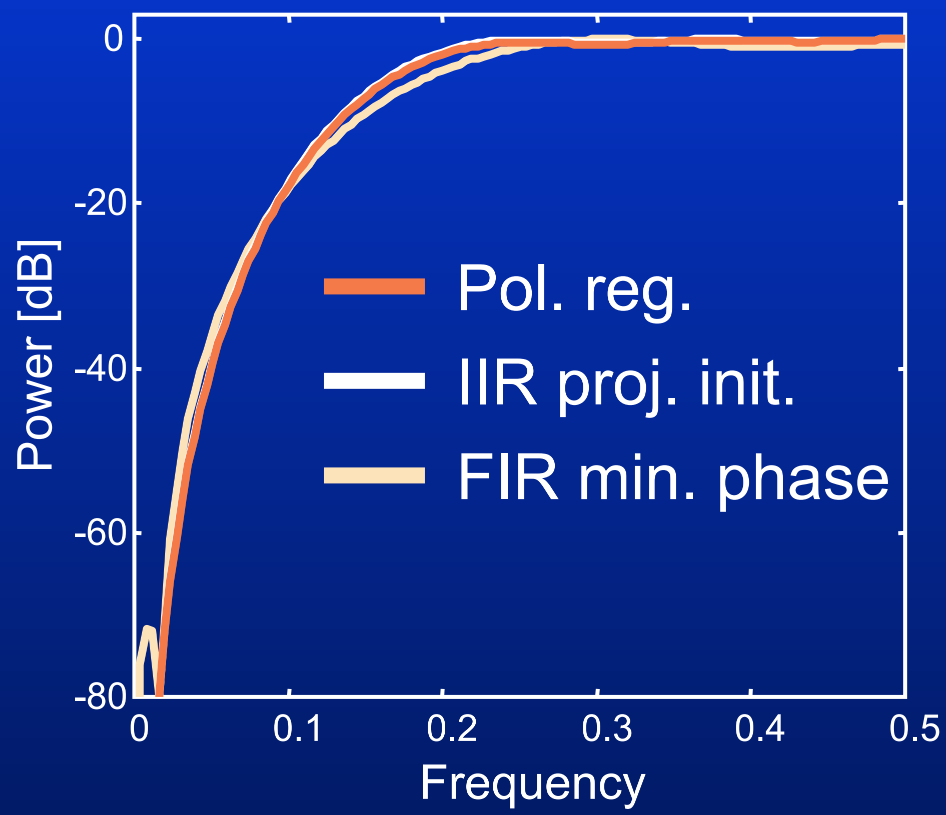
- ML estimator is unbiased, but probably not minimum variance
- Polynomial regression filter showed estimator variance comparable to the ML estimator when above filter cut off frequency
- Polynomial regression filter gave substantial bias for low velocities
- FIR filter give no bias, but significant increase in variance

Optimum clutter filtering

1. Adaptive filters

2. Optimum detection of blood vessel in clutter noise

Frequency Responses



Other quality measures for clutter filters than the frequency response?

Signal Model

- **Signal vector for each sample volume:**

$$\mathbf{x} = [\, x(1), \ldots ,x(N) \,]^T$$

- **Zero mean complex Gaussian process**
- **Three independent signal components:**

Signal = Clutter + White noise + Blood

$$\mathbf{x} = \mathbf{c} + \mathbf{n} + \mathbf{b}$$

- **Signal correlation matrix:**

$$R_x = R_c + \sigma_n^2 I + R_b$$

Optimal Basis of Clutter Space

- Eigenvalue decomposition of the clutter correlation matrix:

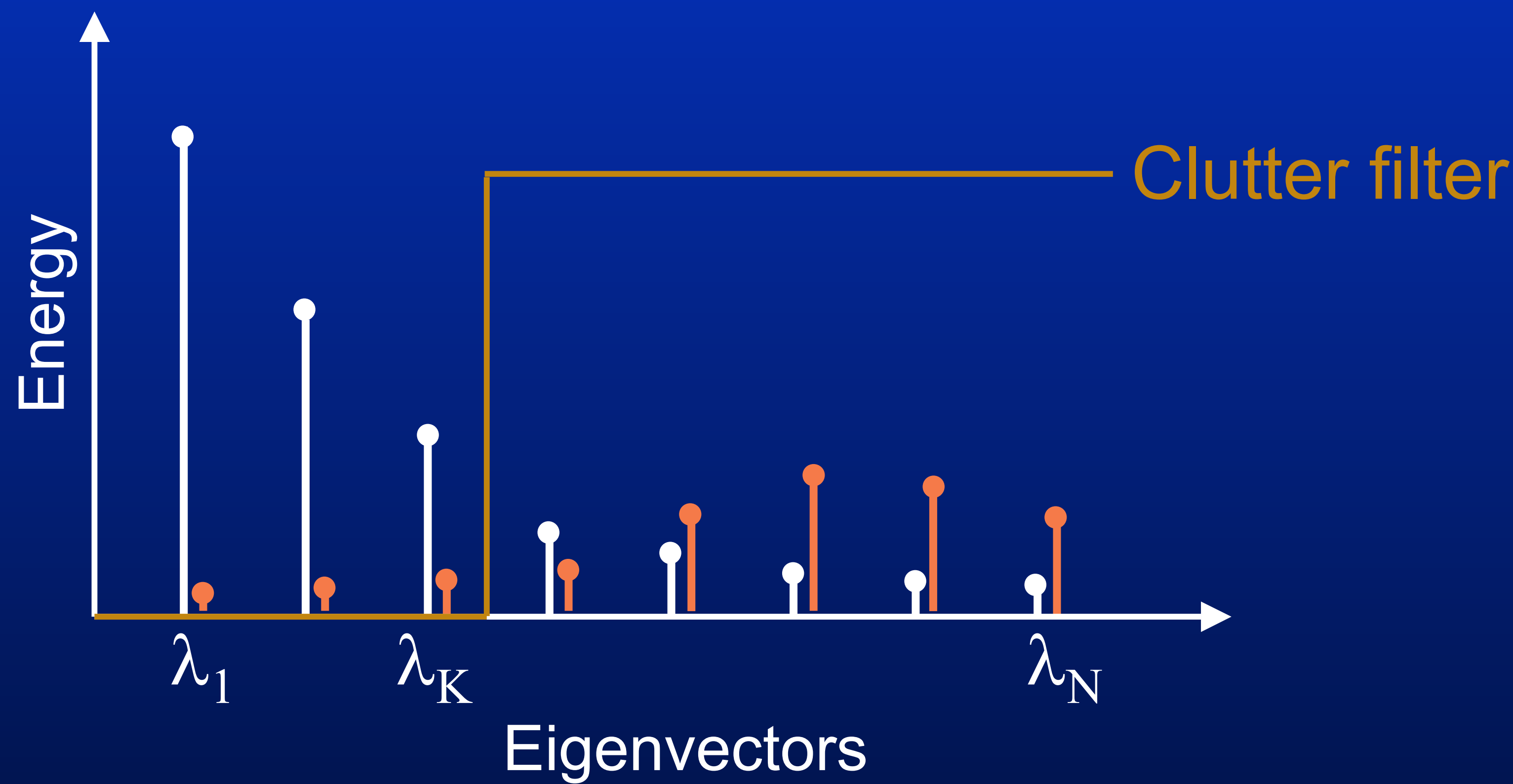
$$\mathbf{R}_c \mathbf{e}_i = \lambda_i \mathbf{e}_i$$

$$\mathbf{R}_c = \sum_{i=1}^{N} \lambda_i \mathbf{e}_i \mathbf{e}_i^H$$

- Use the eigenvectors e_i as a basis for the clutter space (Karhunen-Loeve transform)
- This basis provides maximum energy concentration

Adaptive Regression Filter

Eigenvalue spectrum of clutter + blood



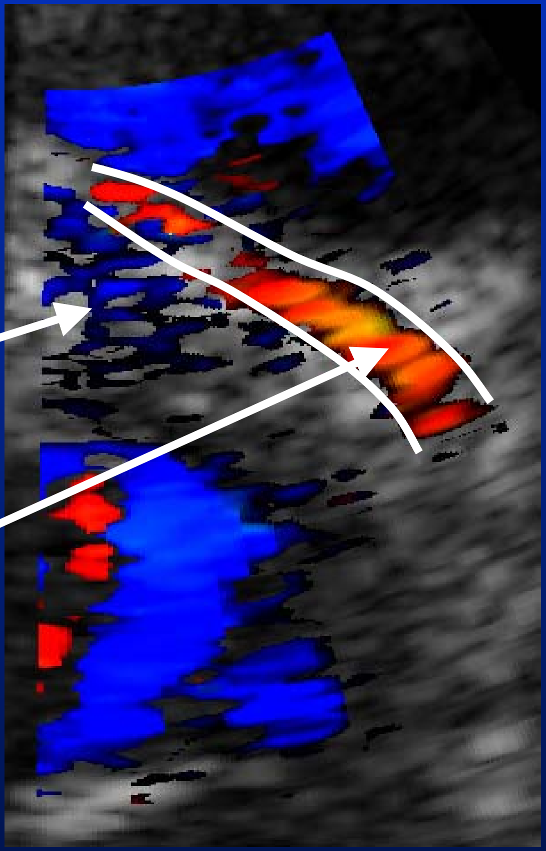
Detection of Blood

A rule for deciding between the two hypotheses:

H_0: No blood is present
H_1: Blood is present

The detector is characterized by

- Probability of false alarm
  $P_F = P(\text{choose } H_1 \mid H_0 \text{ is true})$
- Probability of detection
  $P_D = P(\text{choose } H_1 \mid H_0 \text{ is true})$



Coronary artery

The Optimal Detector I

The Neyman-Pearson lemma:

P_D is maximized under the constraint $P_F \leq \alpha$ by a likelihood ratio test (LRT)

$$L(\mathbf{x}) = \frac{p_{\mathbf{x}|H_1}(\mathbf{x}|H_1)}{p_{\mathbf{x}|H_0}(\mathbf{x}|H_0)} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \gamma$$

The Optimal Detector II

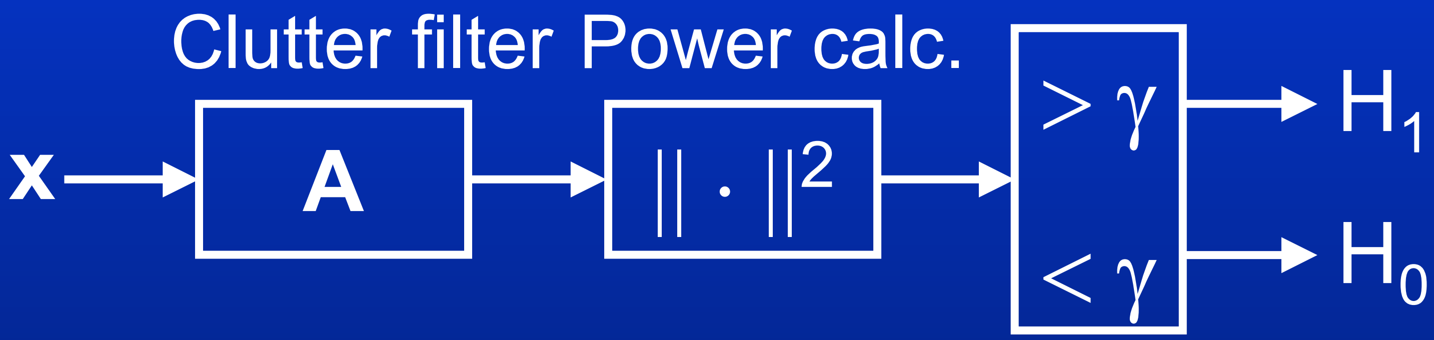
For a Gaussian signal, the LRT can be simplified to:

$$l(\mathbf{x}) = \|\mathbf{A}\mathbf{x}\|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

where $\mathbf{A}=\left(\mathbf{I}-(\mathbf{I}+\mathbf{L})^{-1}\right)^{1/2}\mathbf{E}^{*T}$, and $\mathbf{E}$ and $\mathbf{L}$ are the solution of the generalized eigenvalue problem

$$\mathbf{R}_b\mathbf{E}=\mathbf{R}_{x|H_0}\mathbf{E}\mathbf{L}$$

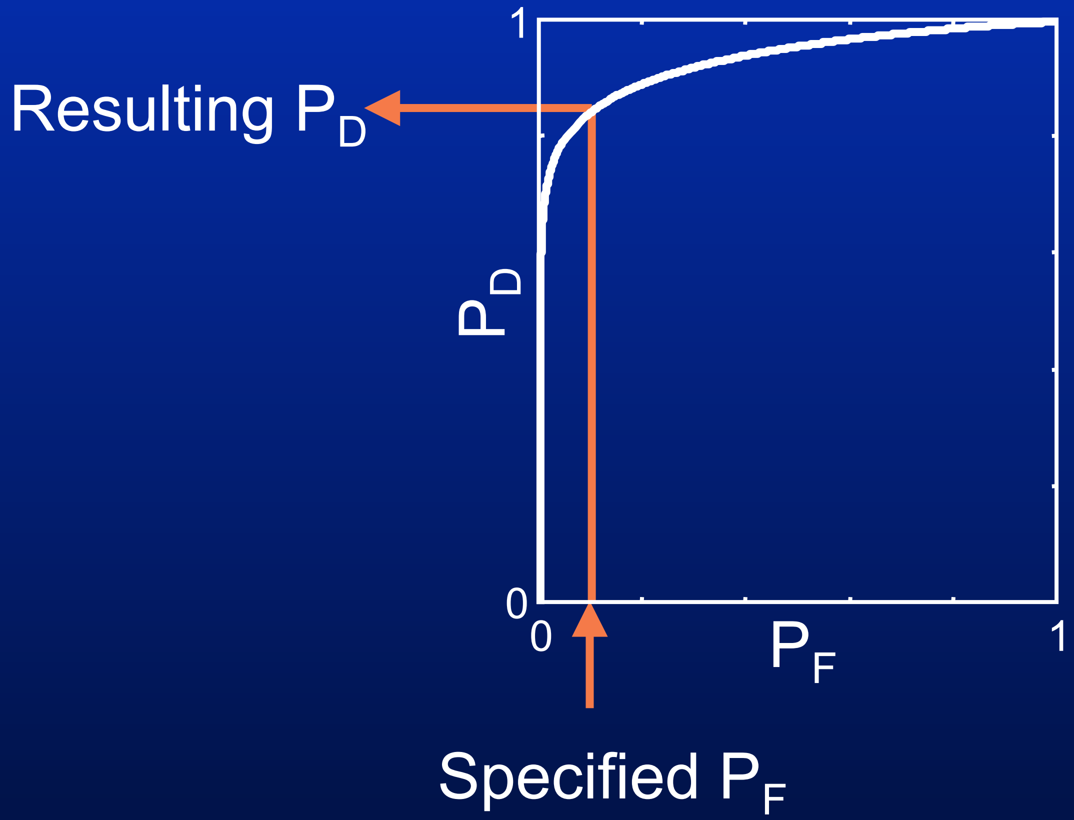
The Optimal Detector III



The same structure as conventional color flow systems, but signal dependent filter matrix **A**

Detector Performance I

- The detection performance is summarized in a receiver operating characteristic (ROC)

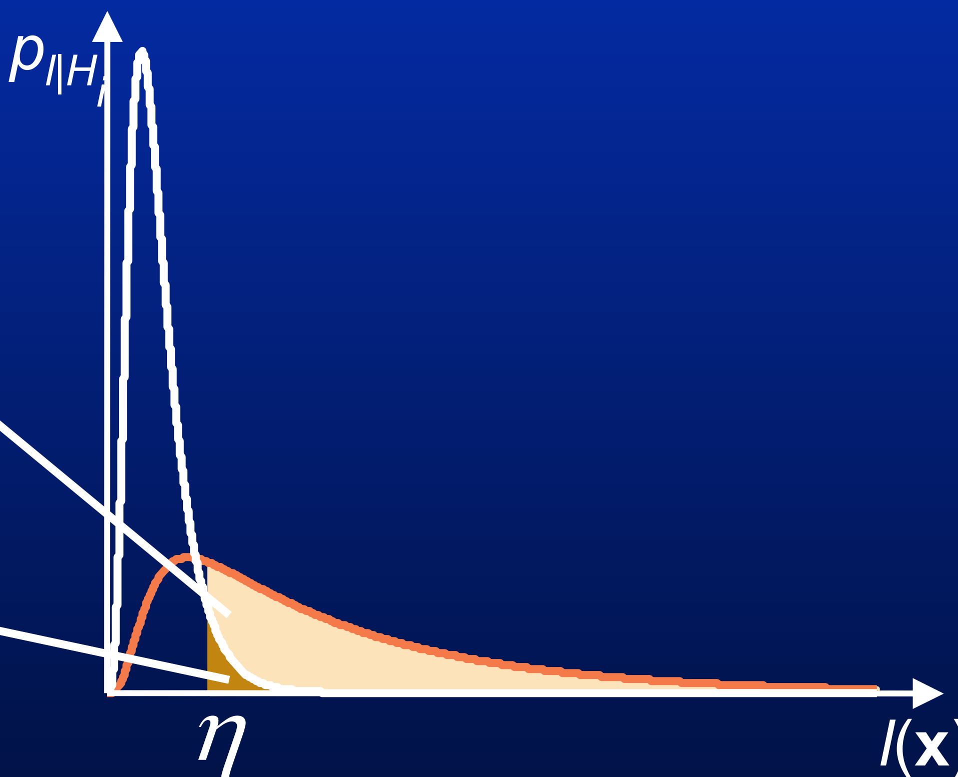


Detector Performance II

- $l(\mathrm{x})$ **is a sum of exponentially distributed variables**
- P_D **and** P_F **is equal to:**

$$P_D = \int_{\eta}^{\infty} p_{l|H_1}(l|H_1)\,dl$$

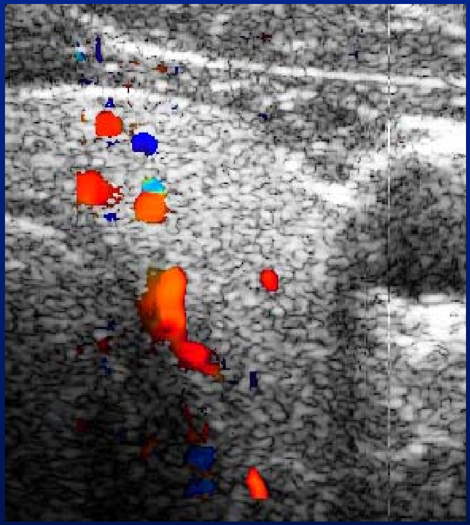
$$P_F = \int_{\eta}^{\infty} p_{l|H_0}(l|H_0)\,dl$$



Data Acquisition

- **Digital RF data recorded with GE Vingmed Ultrasound System Five ultrasound scanner**
- **Complex baseband signals transferred to external computer for processing**

Acquisition parameters	
Center freq.	5.7 MHz
PRF	5 kHz
Rad. samp. freq.	2 MHz
Temp. samples	9



Thyroid gland

Signal Characteristics

- The correlation matrix is estimated by spatial averaging in a region with uniform motion:

$$\hat{\mathbf{R}}_c = \frac{1}{M}\sum_{i=1}^{M}\mathbf{x}_i\mathbf{x}_i^H$$

- Estimated signal parameters:
  - Mean probe movement: 1.0 cm/s
  - Clutter to white noise ratio: 52 dB

- Blood signal:
  - Modeled as a single frequency signal
  - Blood signal to white noise ratio set to 6 dB

ROCs for the Different Filters

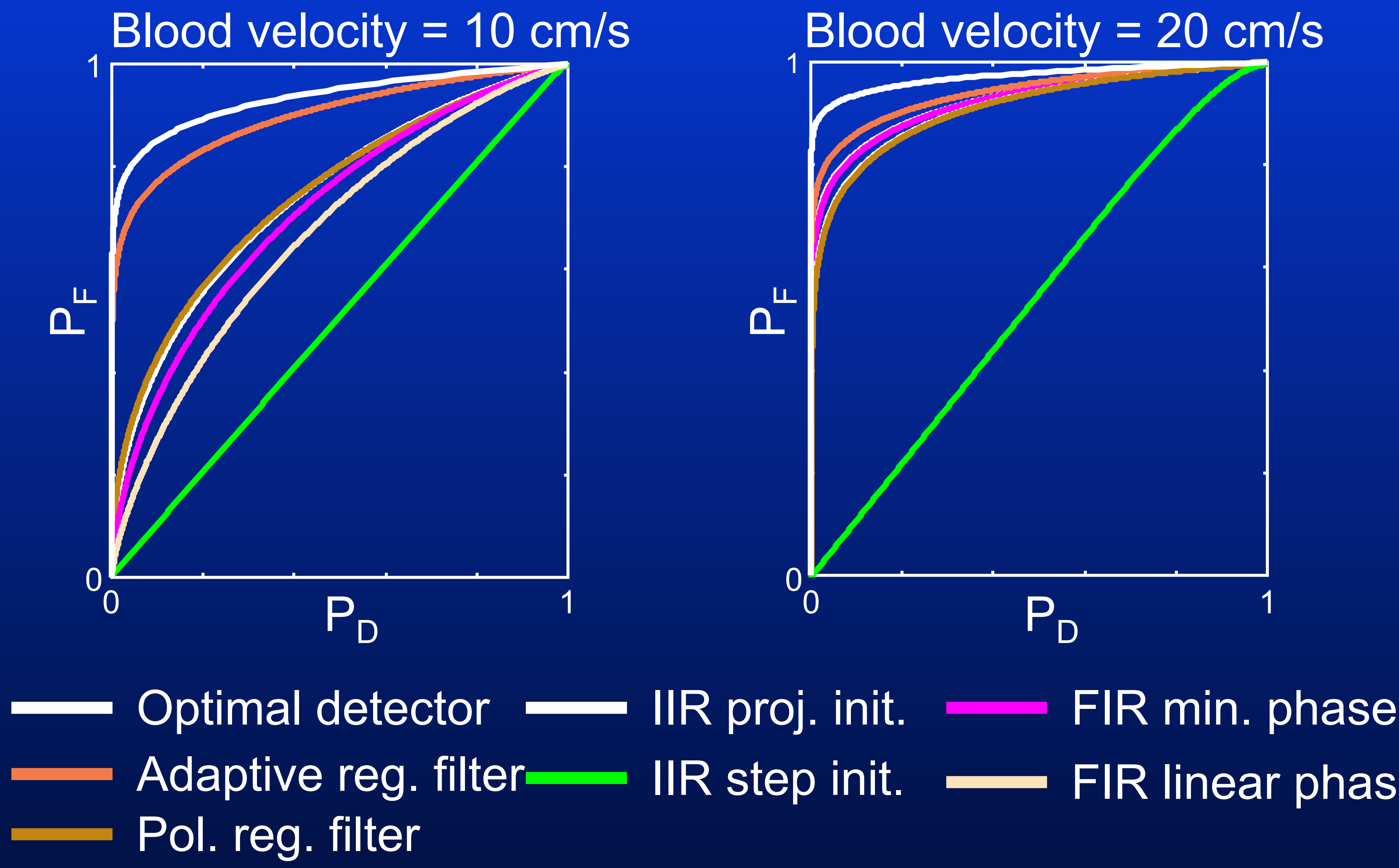
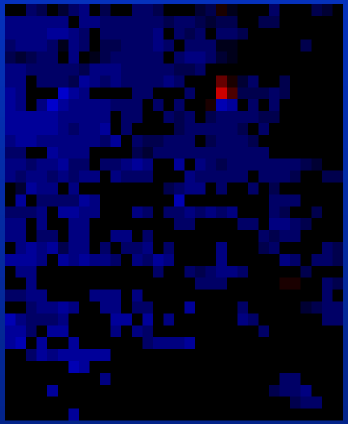


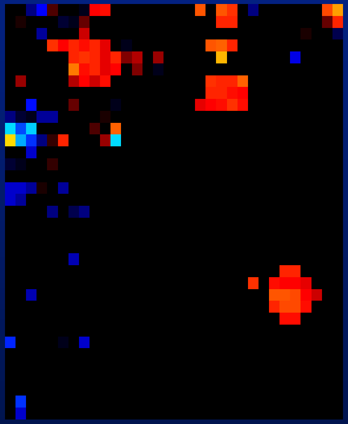
Image Improvement

Example of image improvement with adaptive regression filter

Polynomial regression filter



Adaptive regression filter



Conclusions Clutter filter

- Polynomial regression filters and projection initialized IIR filters have similar performance and are best among the non-adaptive filters
- The computational complexity of the projection initialized IIR filter is equal to the regression filter
- The adaptive regression filter has close to optimum performance

Spatial and temporal averaging

Signal samples from M points in time/space with identical statistical properties

$$x_k = x_k(1),.., x_k(N); k = 1,.., M$$

$$p(x_k) = \frac{1}{\pi^N |C|} e^{-x_k' C^{-1} x_k}$$

$$x_k' C^{-1} x_k = \sum_{n,m=1}^{N} a_{n,m} x_k(n)^* \cdot x_k(m)$$

How to combine these for optimum detection and velocity estimation?

Spatial and temporal averaging

Joint probability function (uncorrelated signals)

$$p(x_1,\ldots x_M)=\prod_{k=1}^{M}\frac{1}{\pi^N|C|}e^{-x_k{}'C^{-1}x_k}$$

Log probability function

$$l(x_1,\ldots x_M)=-\sum_{k=1}^{M}x_k{}'C^{-1}x_k-\pi^{NM}|C|^M$$

$$=-\sum_{n,m}a_{n,m}\sum_{k=1}^{M}x_k(n)^*\cdot x_k(m)-\pi^{NM}|C|^M$$

Spatial and temporal averaging

$$l(x_1,..x_M) = -\sum_{n,m} a_{n,m}\hat{R}_x(n,m) - \pi^{NM}\left|C\right|^M$$

The covariance estimates $$\hat{R}_x(n,m) = \sum_{k=1}^{M} x_k(n)^* \cdot x_k(m)$$

are sufficient statistics for the detection / estimation problem

This means that the optimum detector, as well as min. variance estimators for signal power, mean frequency, and bandwidth, are a function of the covariance estimates

Temporal averaging for mean frequency estimator

$$\hat{R}_k(1) = \sum_{n=1}^{N} x_k(n)^* \cdot x_k(n+1)$$

k=1 : 4, uncorrelated signal packets

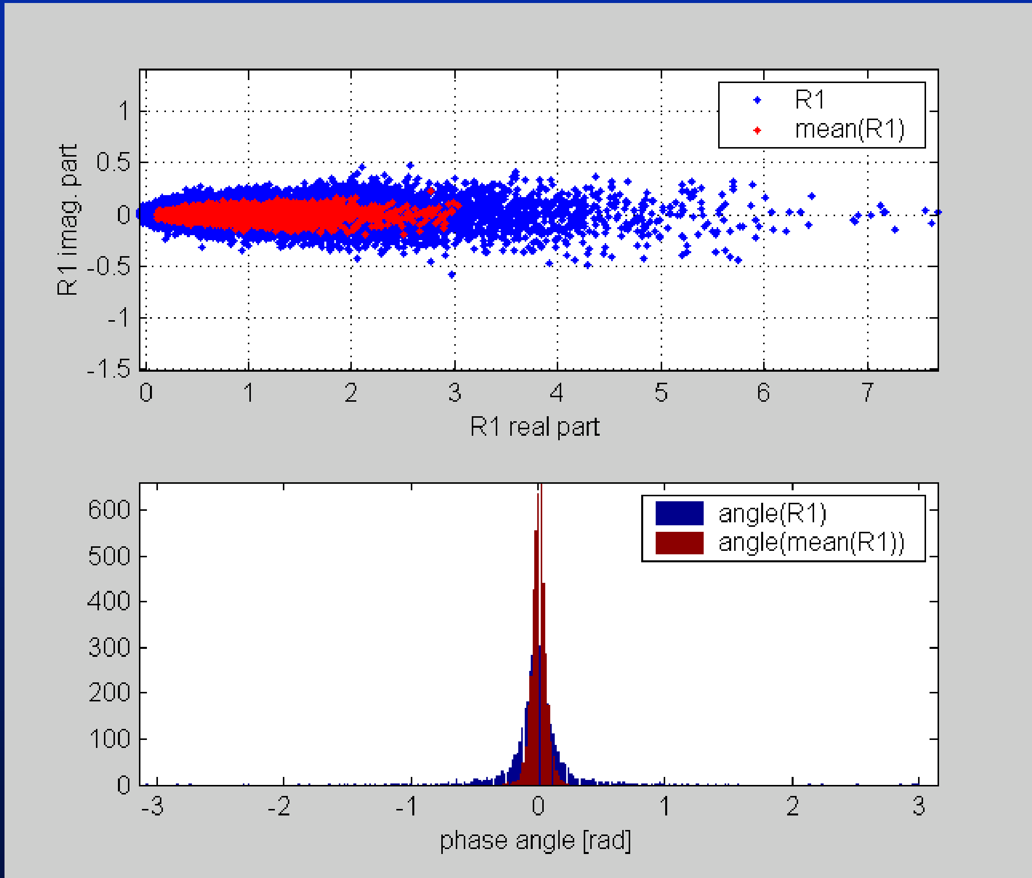
Example:

Effect of averaging R1 :

Variance of angle(R1) reduced by factor 24!

Effect of averaging angle(R1):

Variance reduction factor=4



Spatial/temporal averaging summary

- Efficient variance reduction of signal power, mean frequency and bandwidth can be achieved by averaging data from uncorrelated signal segments.
- Optimum variance reduction is achieved by averaging the complex correlation estimates before calculating the spectral parameters

Further reading/work

Textbooks
Jørgen Arendt Jensen:
***Estimation of Blood Velocities Using Ultrasound, A Signal Processing Approach*,**
Cambridge University Press, 1996. http://www.it.dtu.dk/~jaj/book.html

B.A.J. Angelsen:
Ultrasound Imaging, Waves, Signals, and Signal Processing
http://www.ultrasoundbook.com/

Internet-site for this course:

http://www.ifbt.ntnu.no/~hanst/flowmeas02/index.html