IEEE Ultrasonic symposium 2002

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Short Course 6: Flow Measurements

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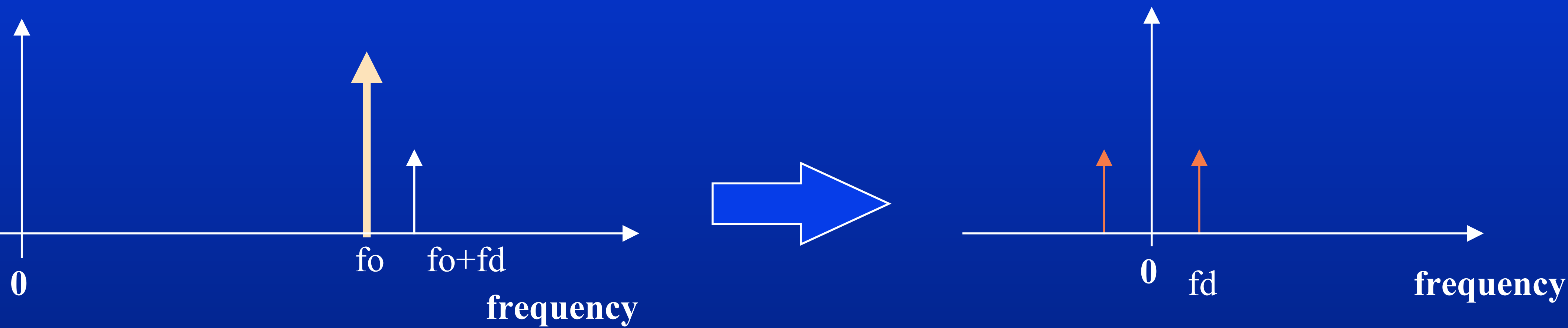
Internet-site for short course:
http://www.ifbt.ntnu.no/~hanst/flowmeas02/index.html

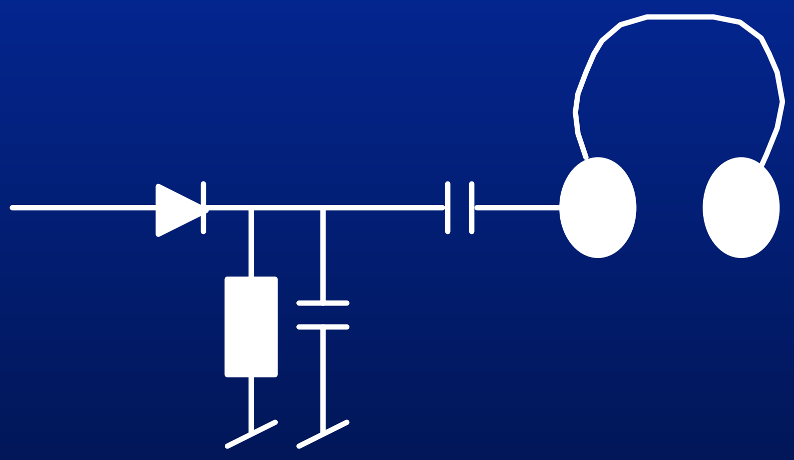
Lecture 2: PW and CW Doppler

Pulsed and continuous wave Doppler

- Dopplershift from moving scatterers
- Stocastic signal model
- Clutter filtering
- Spectrum analysis
- Two dimensional signal model
- Methods to resolve aliasing

Signal processing for CW Doppler





Matlab: cwdoppler.m

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Blood velocity calculated from measured Doppler-shift

$$f_d = 2 f_o \; v \cos(\theta) \; / \; c$$

$$v = c/2f_o/\cos(\theta) \; f_d$$

fd : Dopplershift
fo : Transmitted frequency
v : blood velocity
θ : beam angle
c : speed of sound (1540 m/s)

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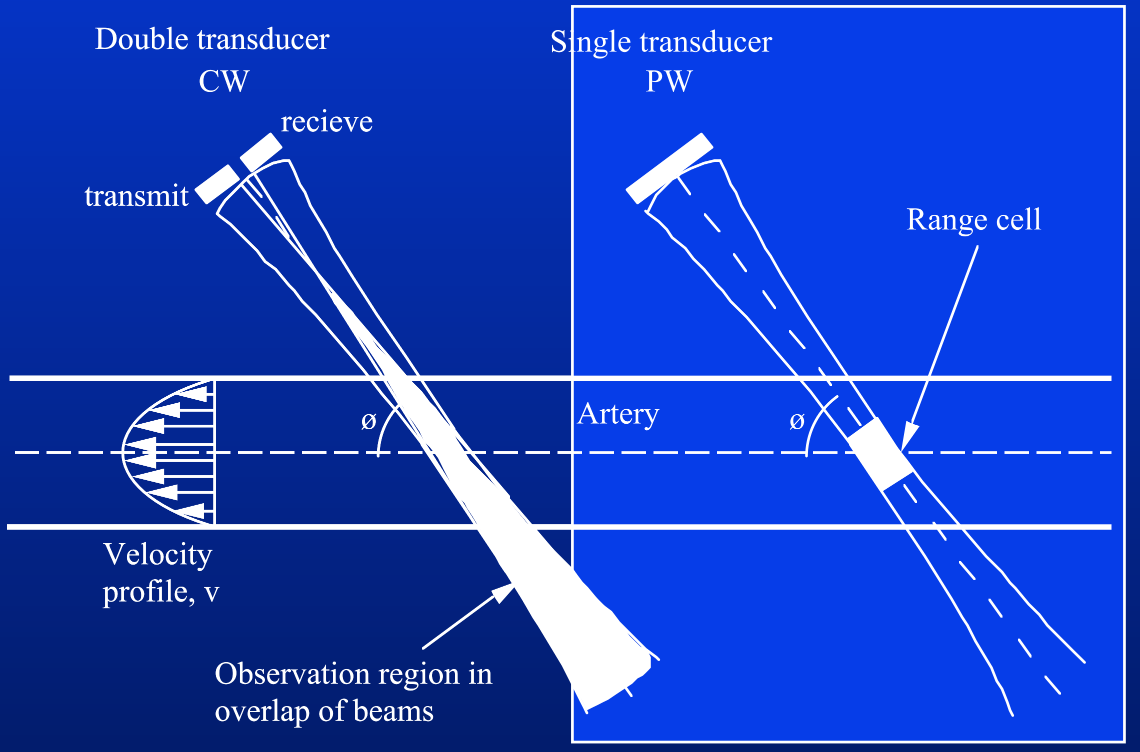
Continous Wave Doppler

Pulsed Wave Doppler



Matlab: pwdoppler.m

Signal from all scatterers within the ultrasound beam

Signal from a limited sample volume

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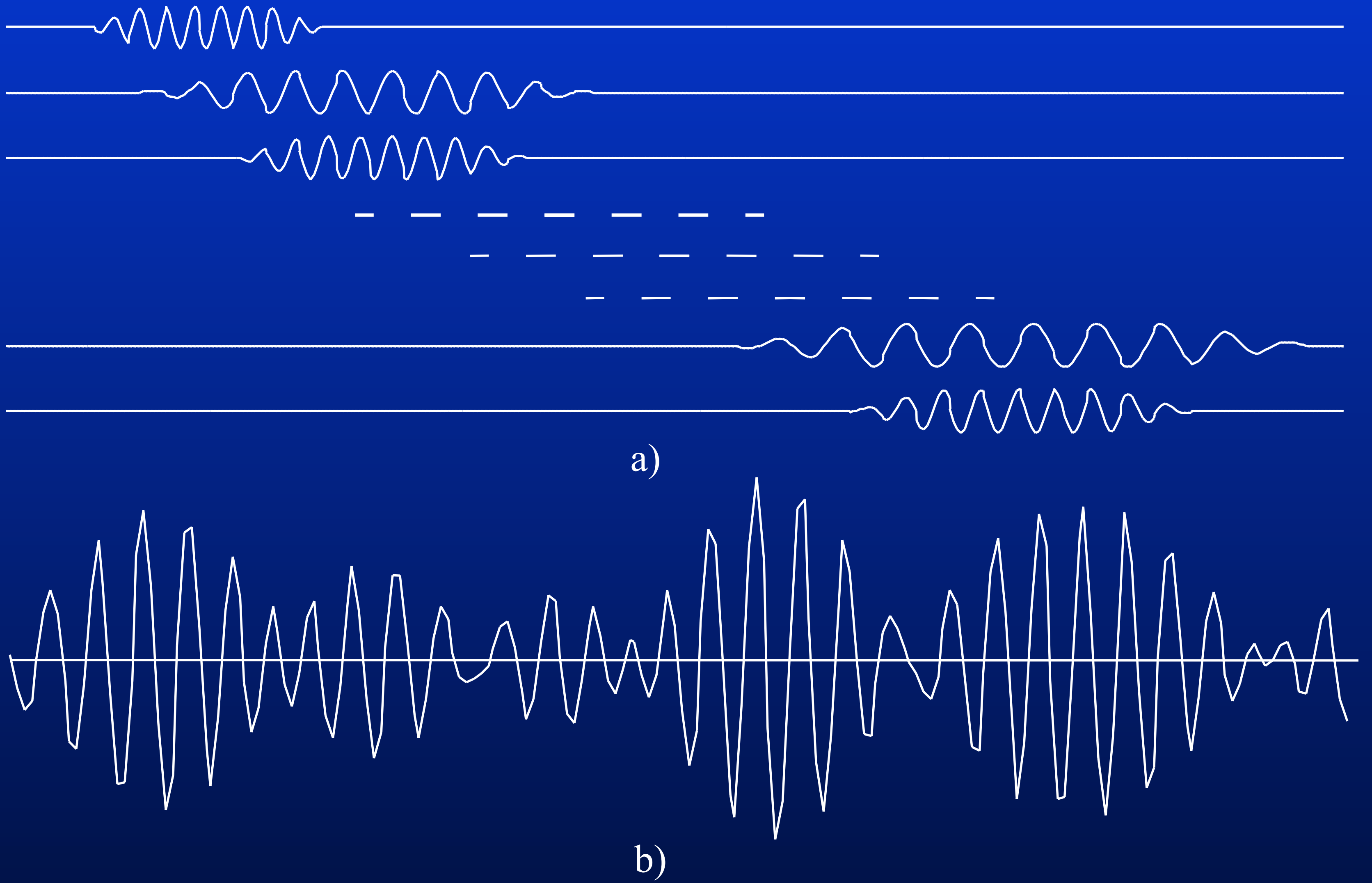
Signal from a large number of red blood cells add up to a Gaussian random process



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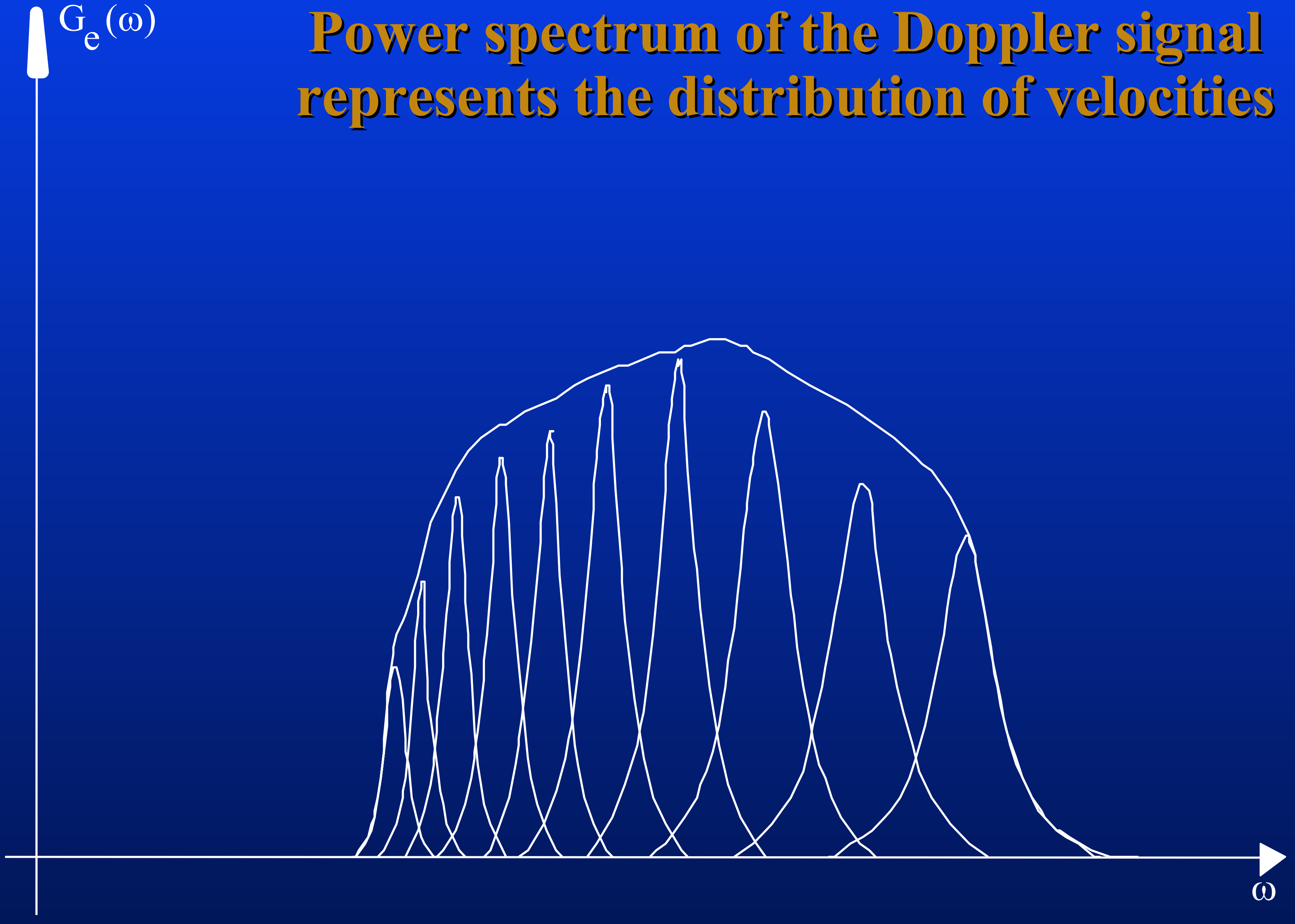
Power spectrum of the Doppler signal represents the distribution of velocities



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Definition of Complex Gaussian process

$$p(\mathbf{z}) = (2\pi)^{-n/2} |\mu|^{-1/2} e^{-\frac{1}{2}\mathrm{z}^{\mathrm{T}}\mu^{-1}\mathbf{z}}$$

signal vector $z = z(1), z(2), \ldots, z(N)$

Covariance matrix $\mu = \langle \mathrm{z}^{\mathrm{T}} z \rangle = \{\langle z(k)^* \cdot z(n) \rangle\}_{k,n}$

$\langle - \rangle$ means expected value (ensemble average)

Note that: $\langle z(k) \cdot z(n) \rangle = 0$

Stationary Complex Gaussian process

Autocorrelation function

$$R(m) \equiv \langle z(n)^* z(n+m) \rangle \quad m = 0, \pm 1, \pm 2, \ldots\ldots$$

Power spectrum

$$G(\omega) \equiv \sum_m R(m) e^{-i\omega m} \quad ; \quad -\pi < \omega < \pi$$

Autocorrelation function
= coeficients in Fourier series of G

$$R(m) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\omega G(\omega) e^{i\omega m}$$

Power spectrum estimate
Statistical properties

Power spectrum estimate:

$$G_N(\omega) = \frac{1}{N}\left|Z_N(\omega)\right|^2$$

$$Z_N(\omega) = \sum_{m=-\infty}^{\infty} w_N(m) z(m) e^{-i\omega m}$$

Expected value:

$$< G_N(\omega) >= \frac{1}{2\pi}\int_{-\pi}^{\pi} d\lambda \left|W(\lambda)\right|^2 G(\omega - \lambda)$$

$$W(\omega) = \sum_m w_N(m) e^{-i\omega m}$$

Power spectrum estimate
Statistical properties

Power spectrum estimate:

$$G_N(\omega) = \frac{1}{N}\left|Z_N(\omega)\right|^2$$

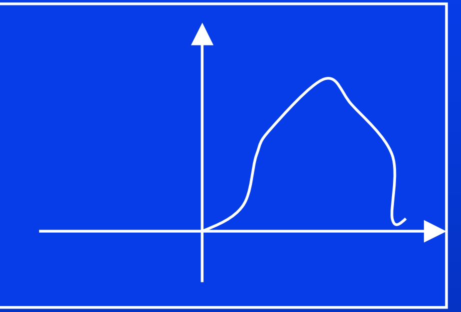
$$Z_N(\omega) = \sum_{m=-\infty}^{\infty} w_N(m) z(m) e^{-i\omega m}$$

Covariance:

$$\operatorname{cov}\left(G_N(\omega), G_N(\omega+\Delta)\right) = \left|\frac{1}{2\pi N}\int_{-\pi}^{\pi} d\lambda W(\lambda) W^*(\lambda-\Delta) G(\omega-\lambda-\Delta)\right|^2$$

$$= \begin{cases} < G_N(\omega) >^2 & \text{when } \Delta = 0 \\ 0 & \text{when } \left|\Delta\right| > 1/N \end{cases}$$

Computer simulation of Complex Gaussian process



1. Complex Gaussian white noise Zn(0),..,Zn(N-1)
2. Shape with requested power spectrum: $Z(k)=\sqrt{G(2\pi k/N)}\, Zn(k)$; k=0,..,N-1
3. Inverse FFT: z(n) = ifft(Z)

$$< |Z(w)|^2 >= G(w)|Zn(w)|^2 = G(w); \text{ for } w= 2\pi k/N$$

Power spectrum for z(n):
(smoothed version of G(w))

$$G_Z(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\lambda \left|W(\lambda)\right|^2 G(\omega-\lambda)$$

$$W(\omega) = \sum_{m=0}^{N-1} e^{-i\omega m}$$

Autocorrelation function:

$$R_z(m) = Tri(m)\cdot R(m)$$

Computer simulation of Complex Gaussian process

- The power spectrum of the simulated signal is smoothed with a window given by the number of samples N
- The autocorrelation function of the simulated signal

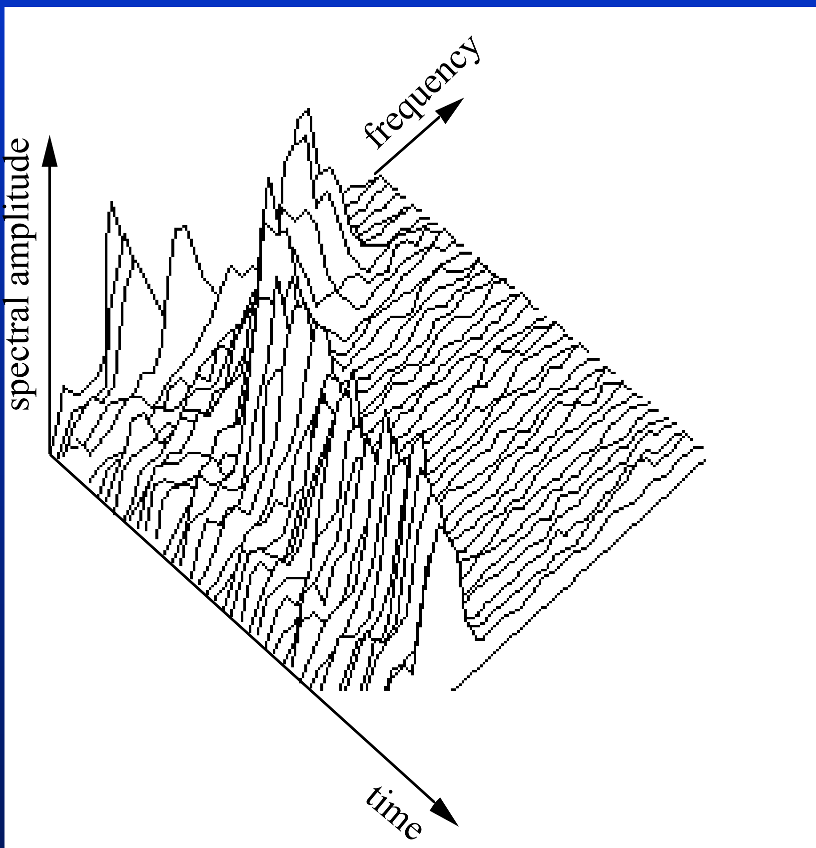
  $R_z(m) = 0$ for $m > |N|$

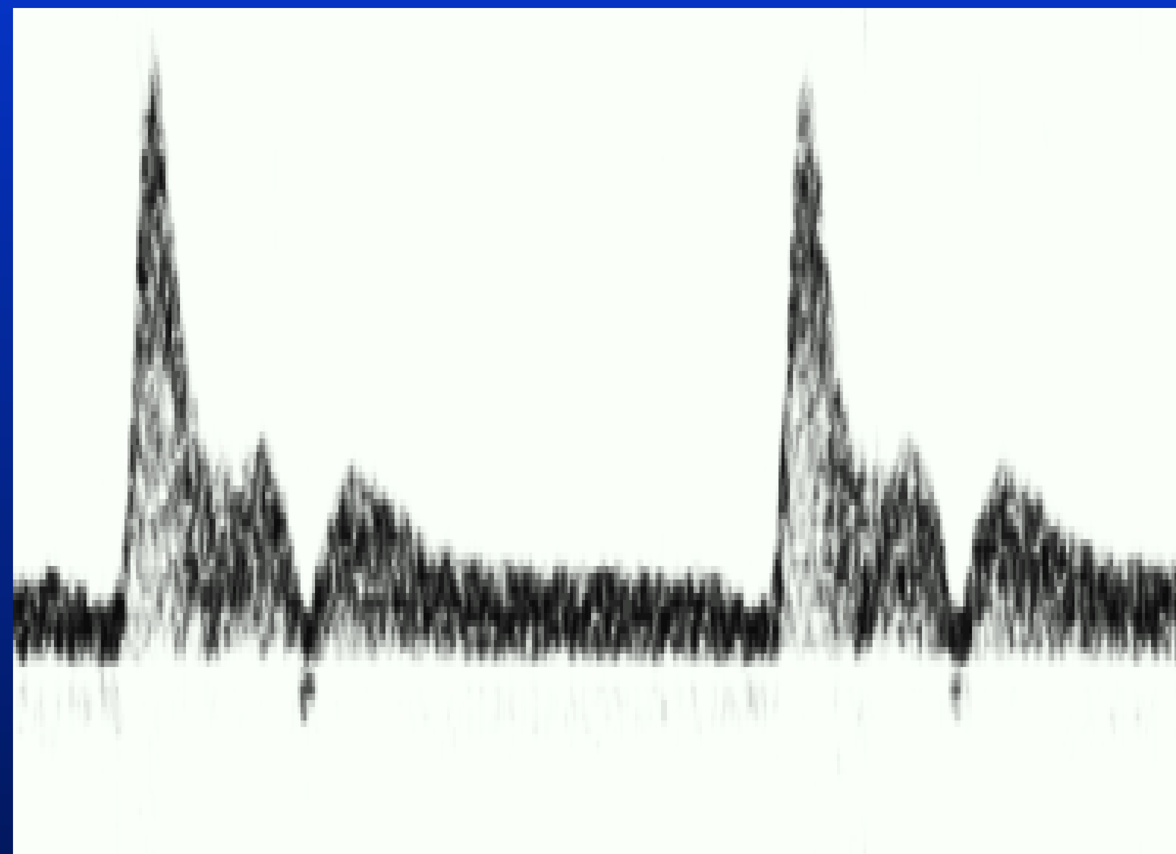
Matlab: CsignalDemo.m

Properties of power spectrum estimate

- Fractional variance = 1 independent of the window form and size
- $G_N(\omega_1)$ and $G_N(\omega_2)$ are uncorrelated when $|\omega_1 - \omega_2| > 1/N$
- Increasing window length N gives better frequency resolution, but no decrease in variance
- Smooth window functions give lower side lobe level, but wider main lobe than the rectangular window
- Decrease in variance can be obtained by averaging spectral estimates from different data segments.

Doppler spektrum





AoDopplerStenose.au

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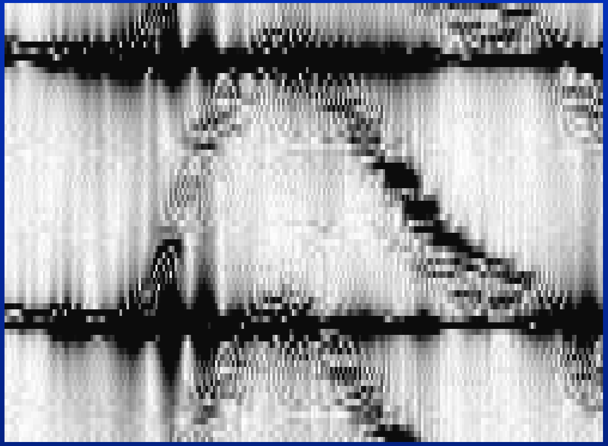
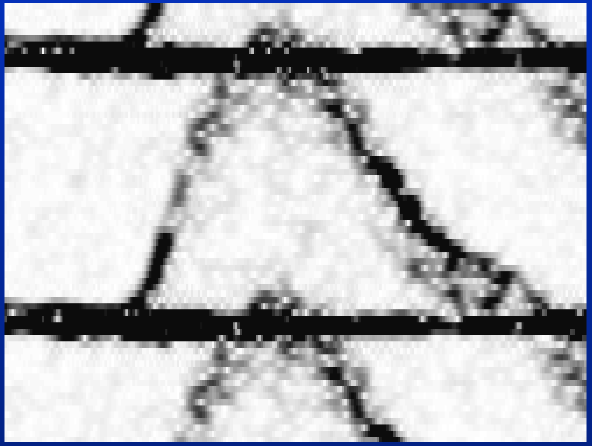
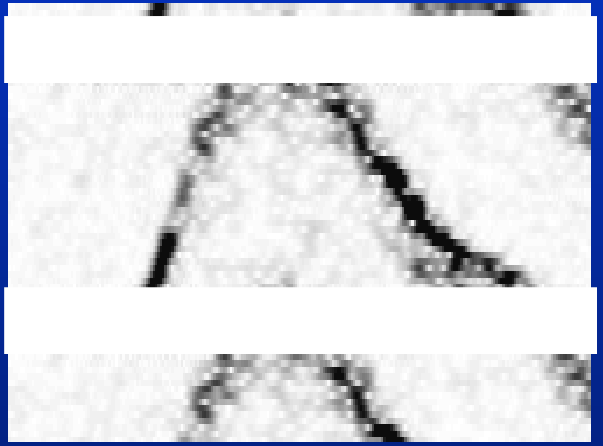
Clutter noise in spectral Doppler



Rectangular window



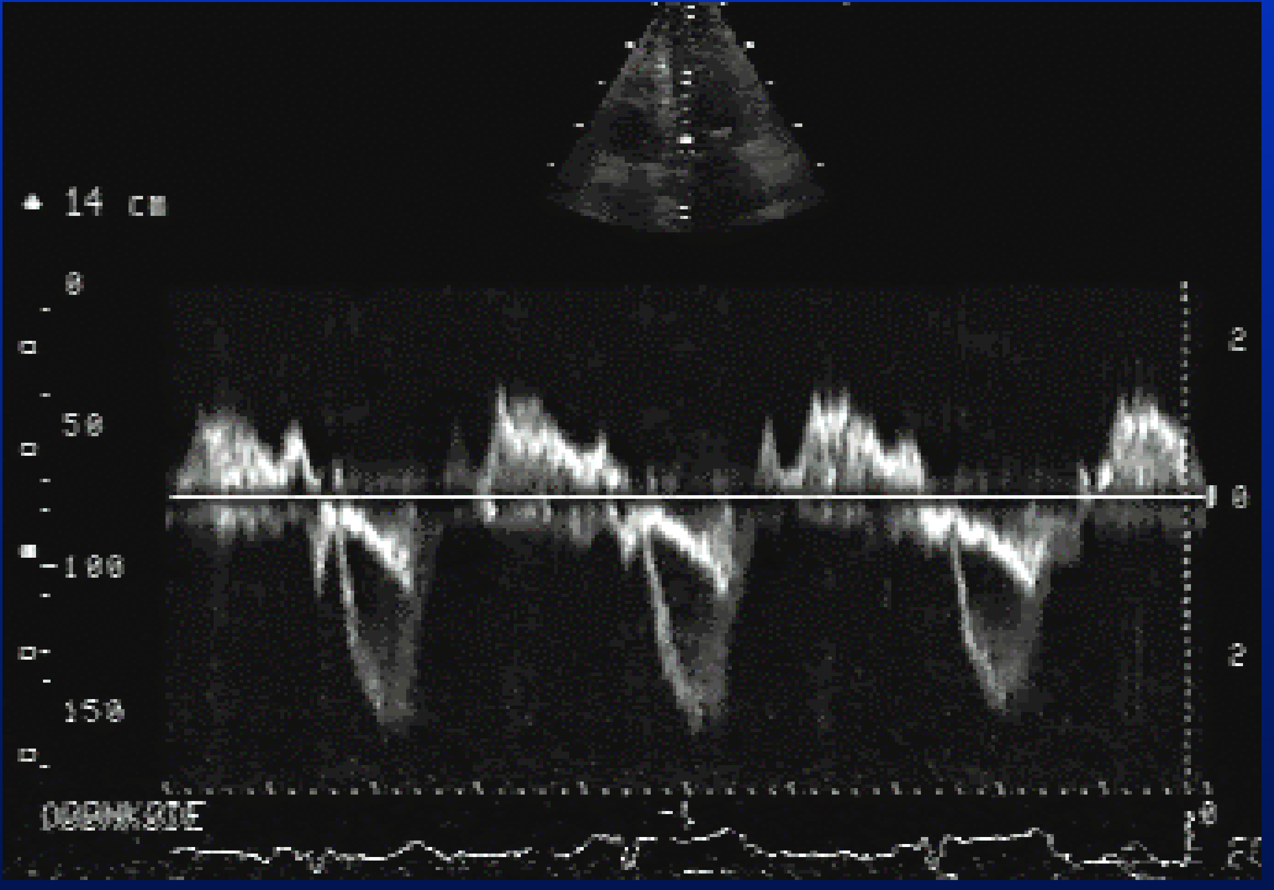
Hamming window



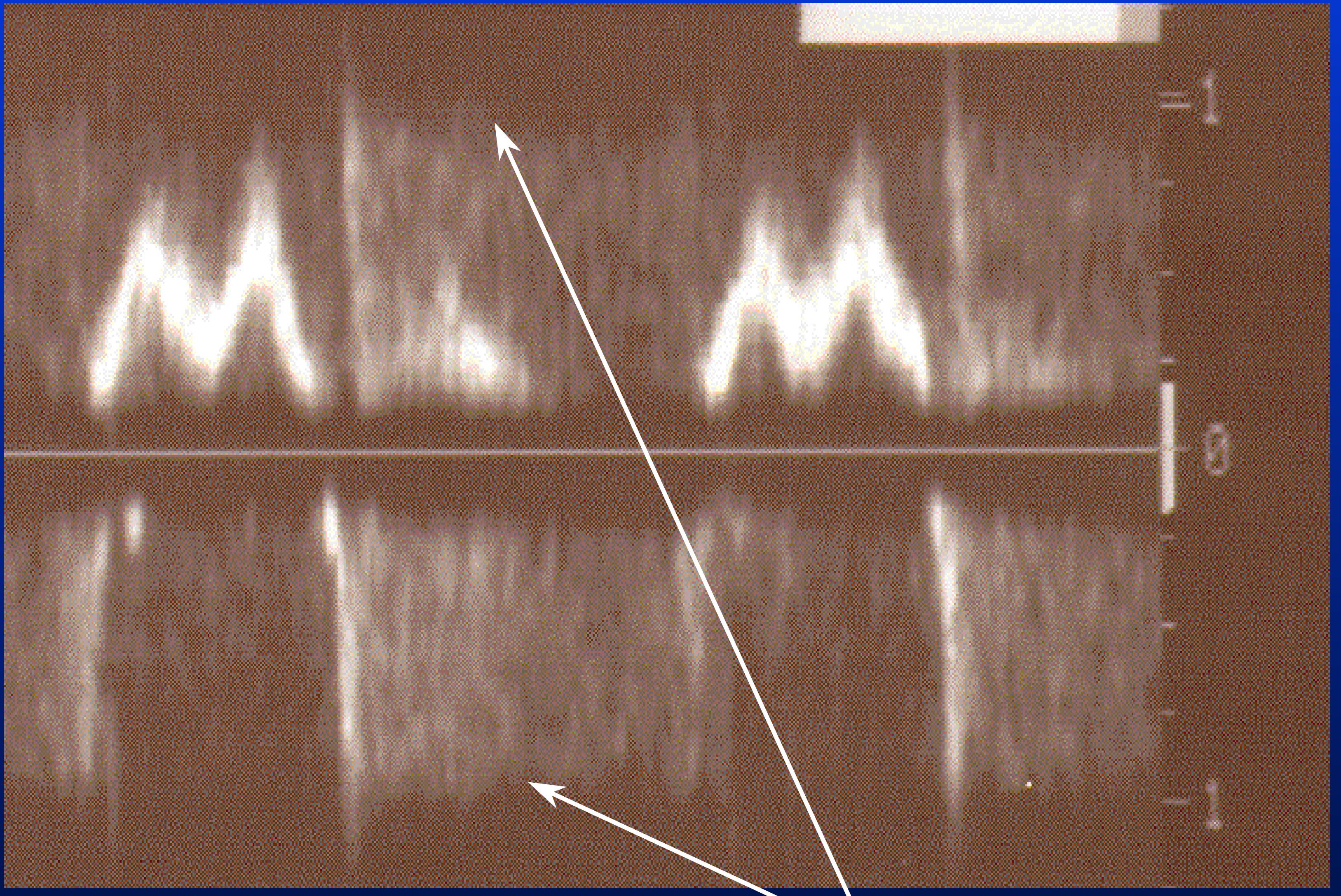
High pass filter

Matlab: Dopplerspectrum.m

Doppler spectrum analysis:
Different velocities separated in frequency

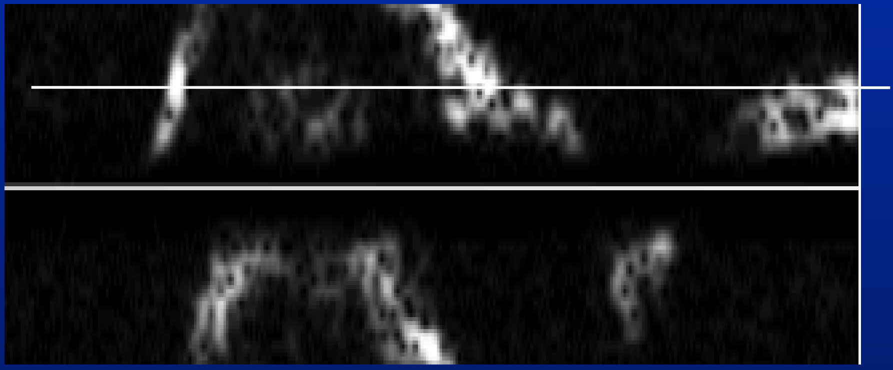


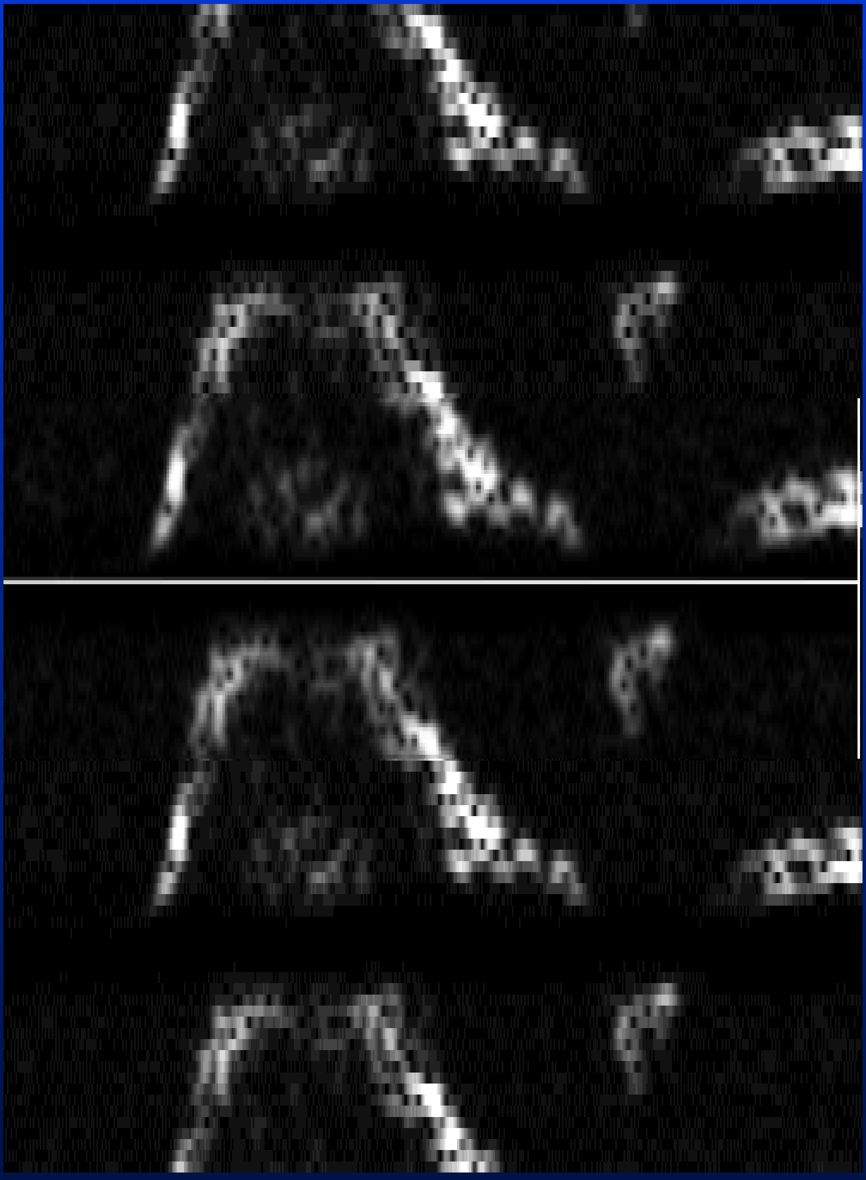
Nyquist limit in Pulsed wave Doppler



Nyquist-limit

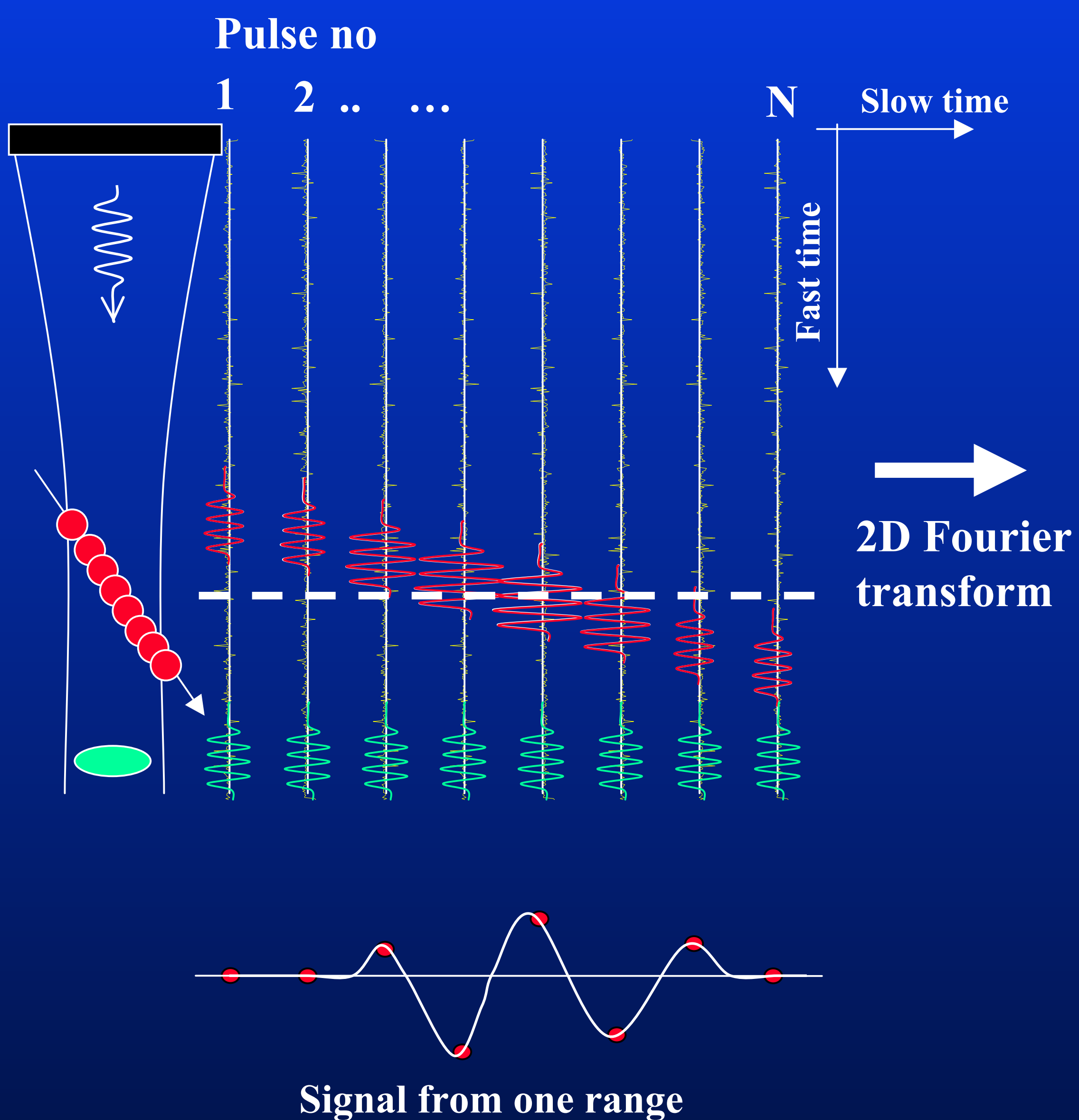
Doppler spectrum
Subclavian Artery

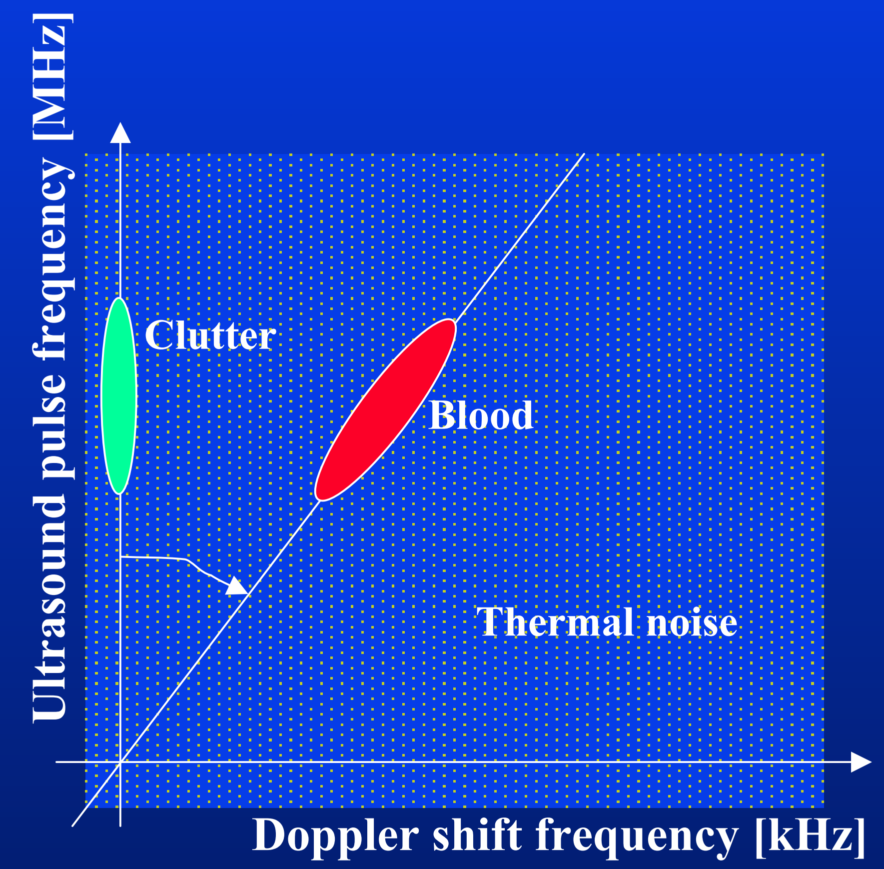


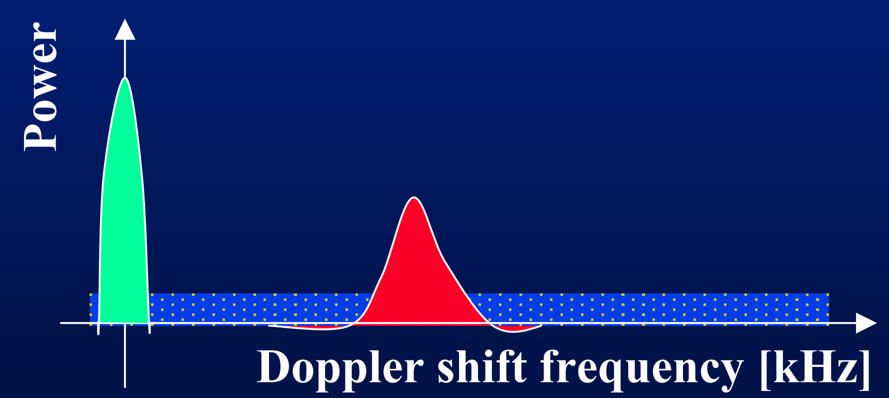


Velocity waveform restored by stacking

Signal from moving scatterer







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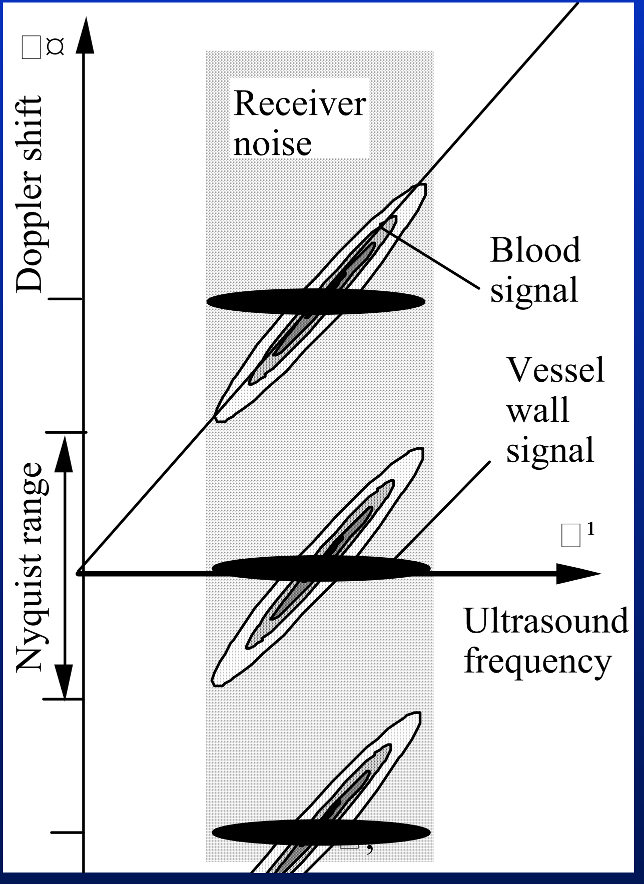
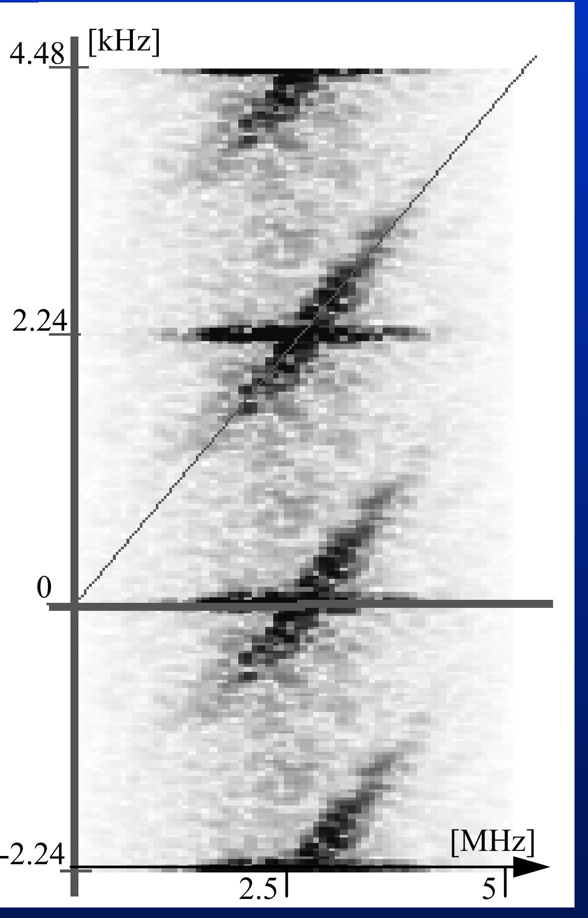
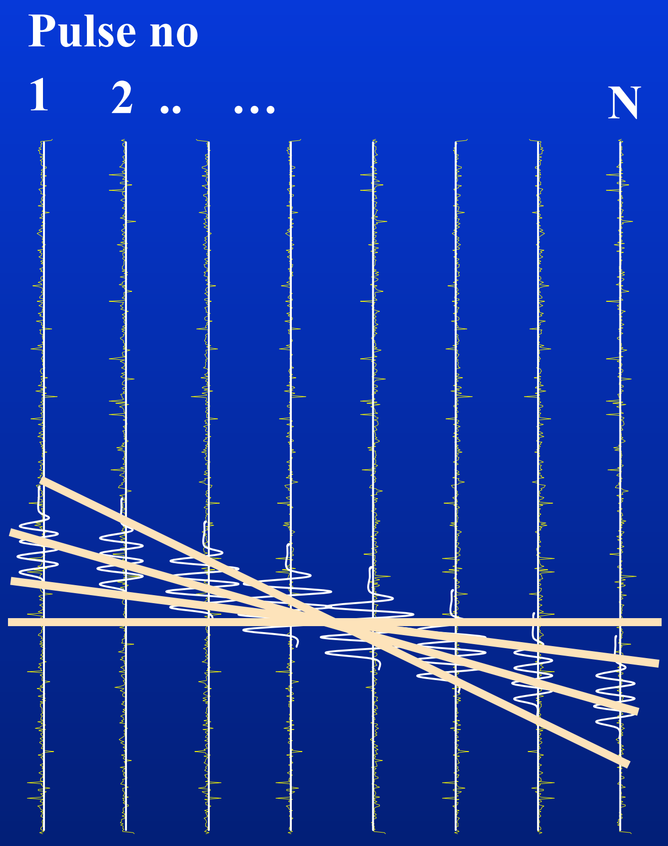
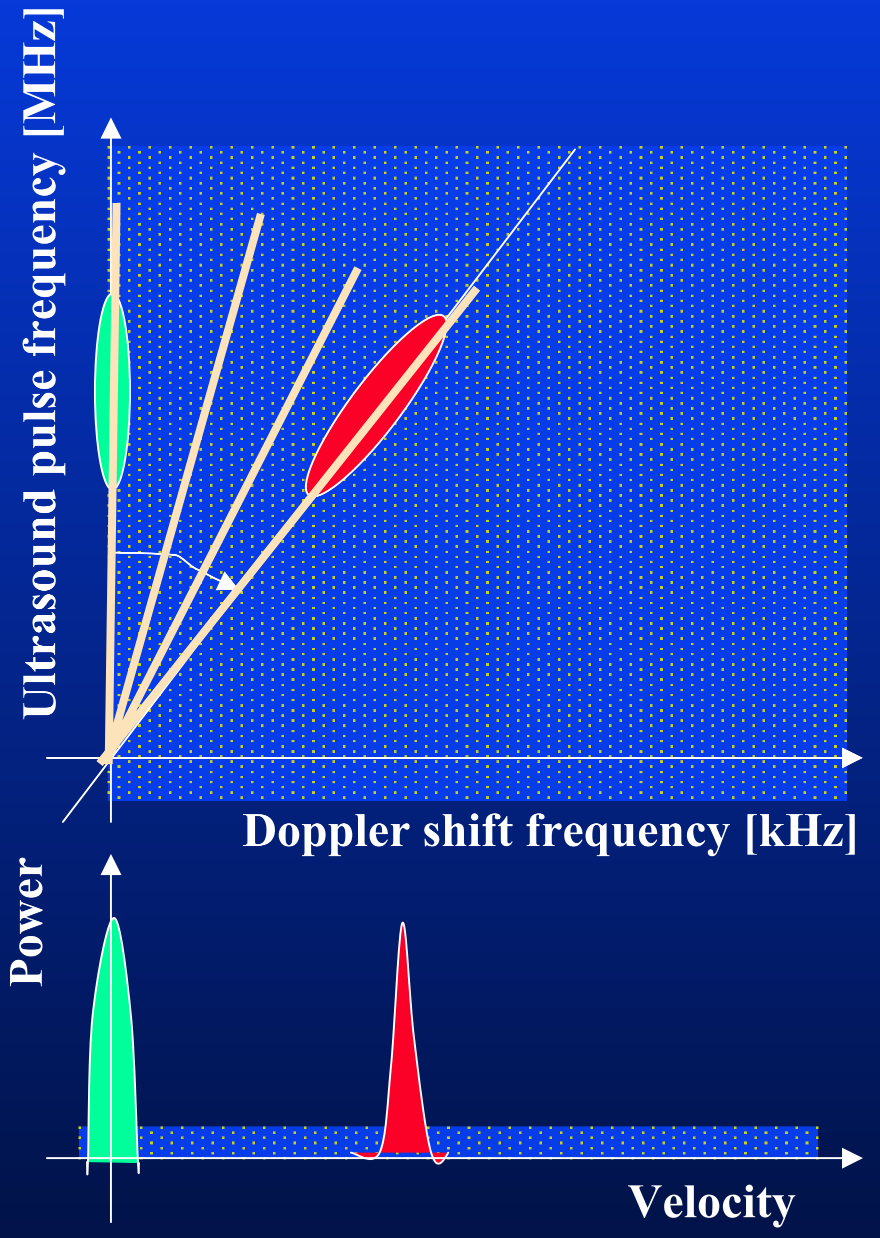
2D Spectrum Subclavian artery





Velocity matched spectrum algorithm





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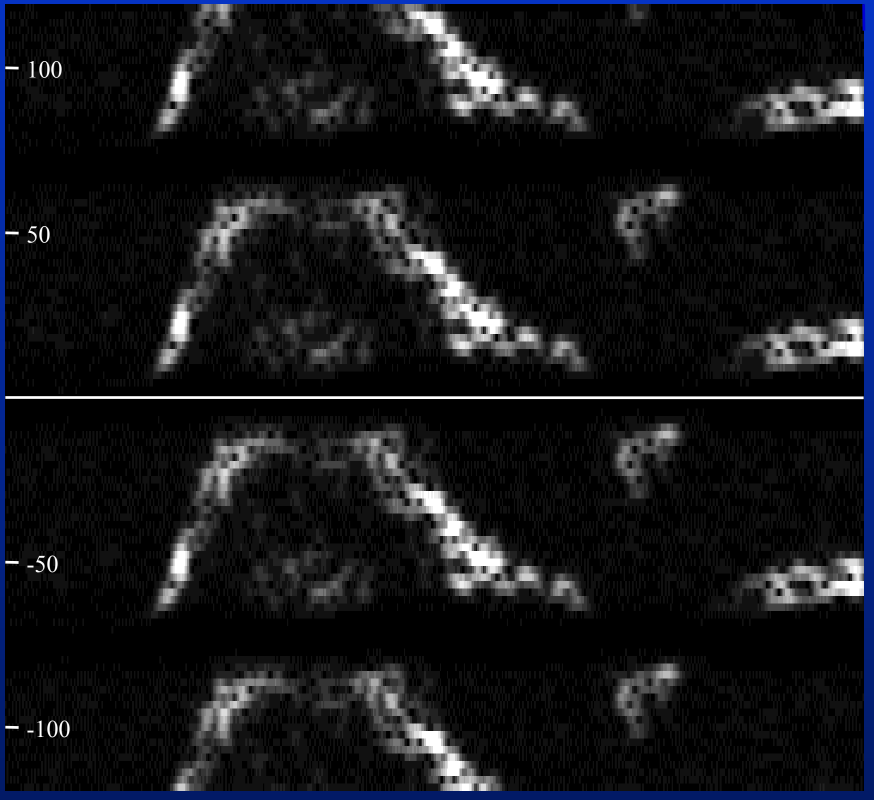
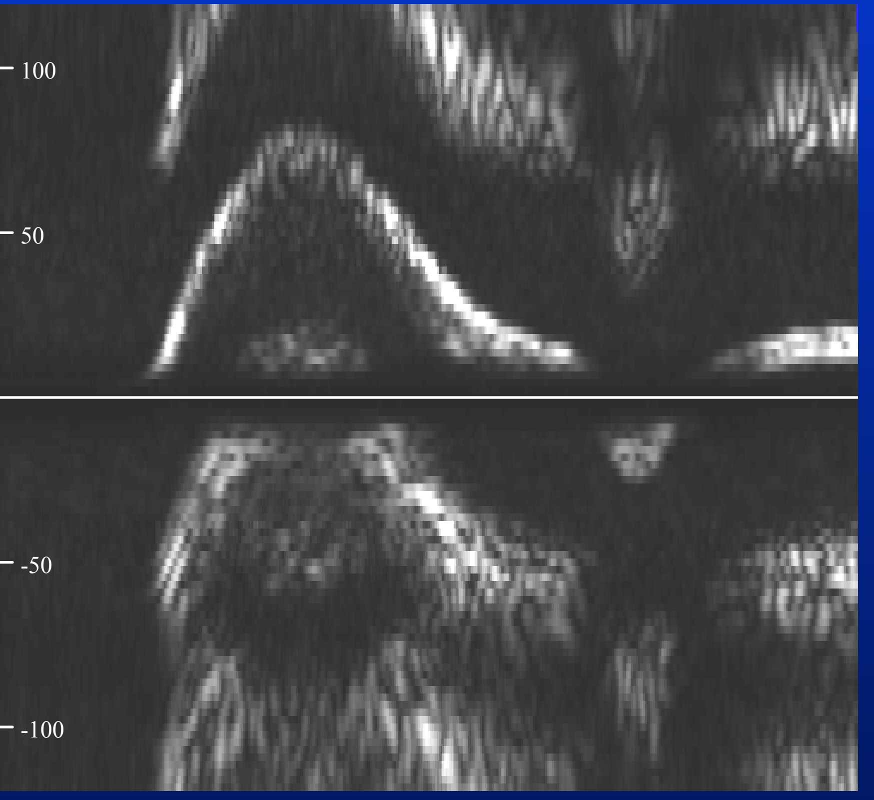
Blood velocity spectrum
Subclavian Artery



Conventional spectrum



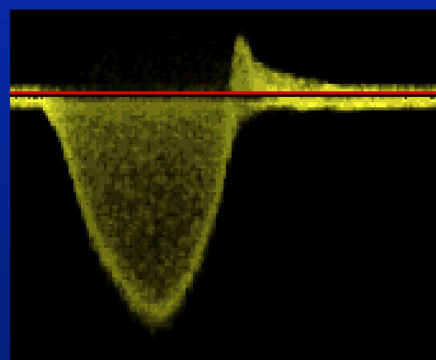
Velocity matched spectrum

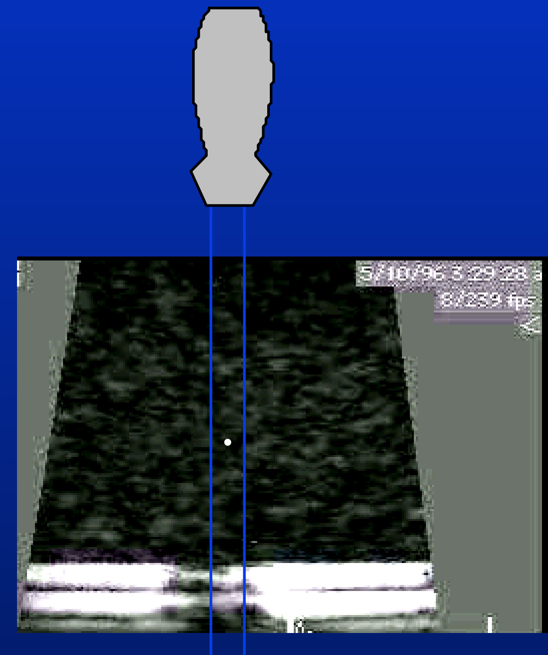
CW (Continuos Wave) Doppler

Tid

Velocity

Doppler frequency





Doppler spectrum contains velocities from the entire beam

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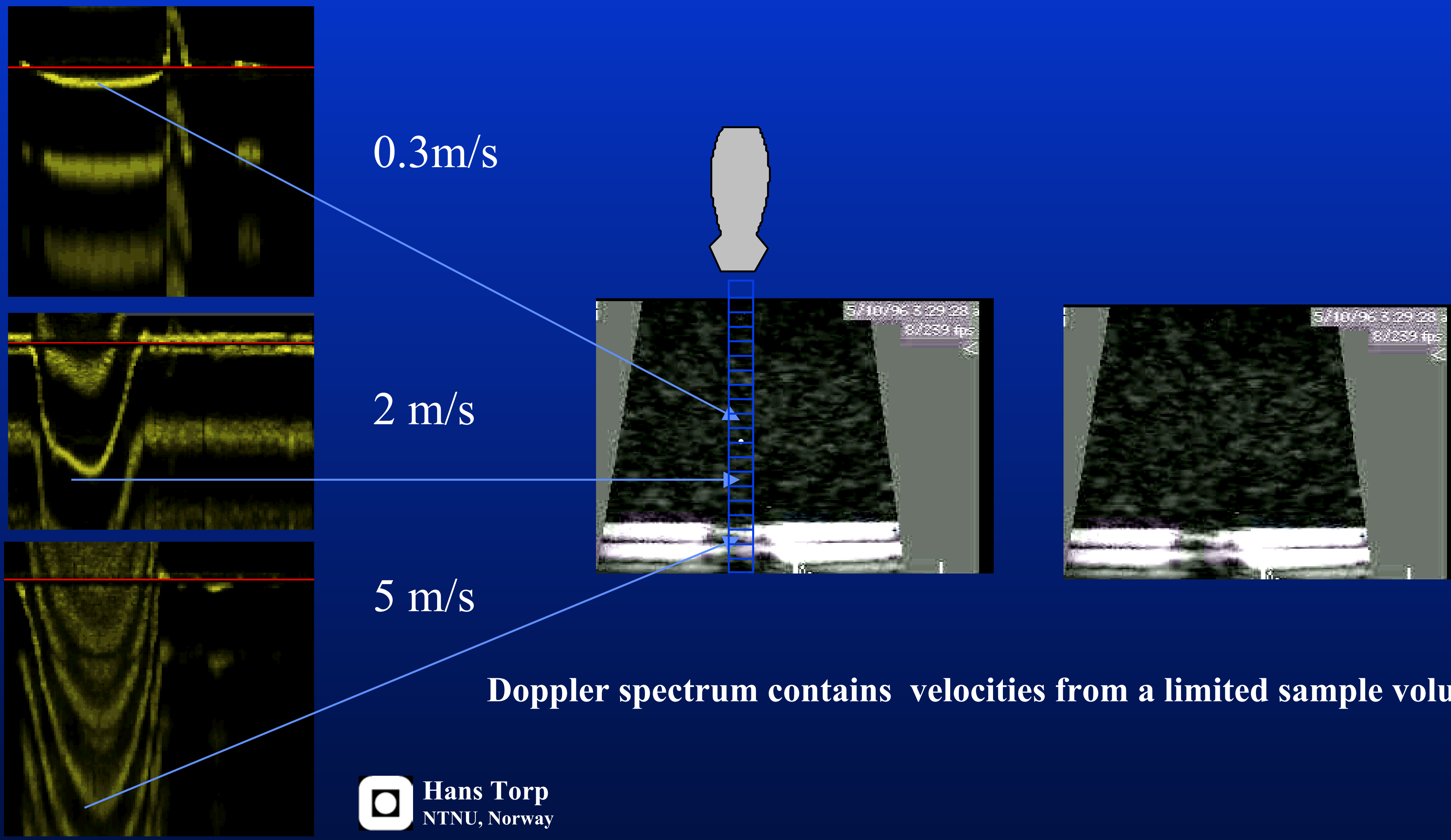
PW (pulsed Wave) Doppler



Doppler spectrum contains velocities from a limited sample volume

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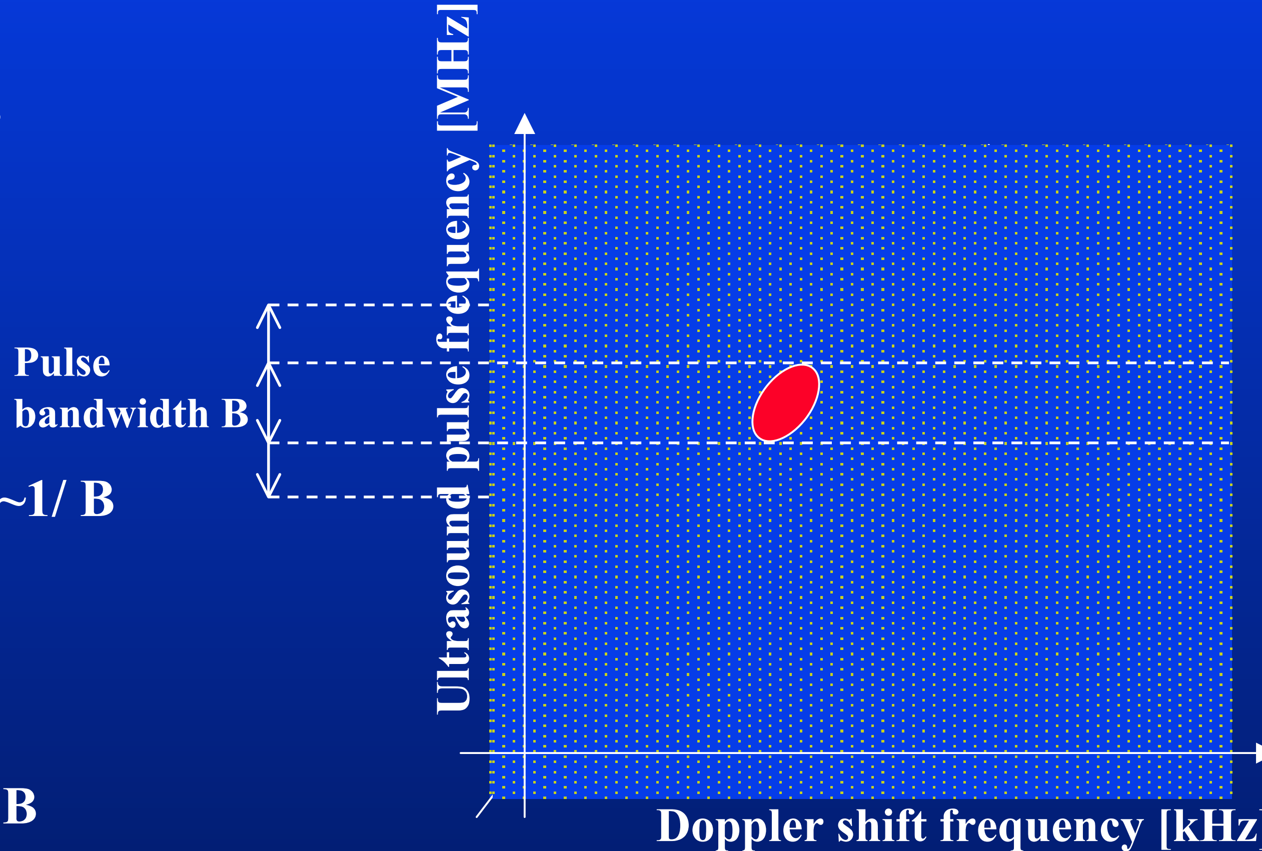
SNR considerations PW Doppler

What is the best pulse length?

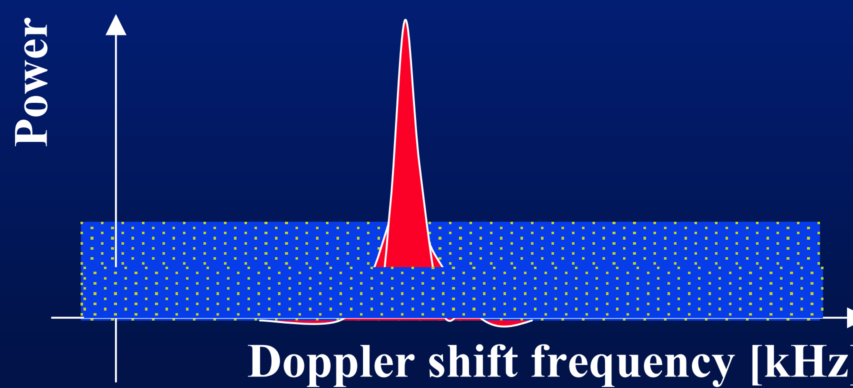
Energy per pulse limited by thermal index or ISPTA



Sample vol. size ~ pulse length ~1/ B

Thermal noise level ~ B
(matched filter)

Blood signal spectral peak ~1/ B
(blood vessel larger than sample vol.)



Spectral SNR ~ 1/ B^2

Double pulse length give + 6 dB SNR

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Summary spectral Doppler

- Complex demodulation give direction information of blood flow
- Smooth window function removes sidelobes from clutter-signal
- PW Doppler suffers from aliasing in many cardiac applications
- Aliasing can be resolved by velocity matched spectrum, if laminar flow,and small angle between beam and velocity vector
- SNR increases by the square of pulse length in PW Doppler